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Product Innovation with Lumpy Investment

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Abstract

This paper considers a firm that has the option to undertake product innovations. For each product innovation the firm has to install a new production plant. We find that investments are larger and occur in a later stadium when more of the old capital stock needs to be scrapped. Moreover, we obtain that the firm's investments increase when the technology produces more profitable products. We see that the firm in the beginning of the planning period adopts new technologies faster as time proceeds, but later on the opposite happens. Furthermore, we find that the firm does not invest such that marginal profit is zero, but instead marginal profit is negative. Moreover, we find that if the time it takes to double the efficiency of technology is larger than the time it takes for the capital stock to depreciate, the firm undertakes an initial investment. Finally, we show that when demand decreases over time and when fixed investment cost is higher, that the firm invests less throughout the planning period, the time between two investments increases and that the first investment is delayed.

Key words: Impulse Control Maximum Principle, Optimal Control, discrete continuous system, state-jumps, product innovation, retrofitting

JEL-codes: C61, D90, 032, 033

1 Introduction

In today's knowledge economy innovation is of prime importance. Innovation has led to the extraordinary productivity gains in the 1990's . In current business practice it is felt that the heat is on and that firms must innovate faster just to stand still (The Economist, October 13th 2007, Innovation: Something new under the sun). Therefore, technological progress is a crucial input

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for firms in taking their investment decisions. Greenwood et al. (1997) argue that technological progress is the main driver of economic growth. They discovered that in the post-war period in the US about 60% of labor productivity growth was investment specific. Yorokoglu (1998) notes that information technology is a prime example where embodied technological progress led to an improvement of computing technology on the order of 20 times within less than a decade in the 1980's-90's.

This paper combines technology adoption with capital accumulation, taking into account technological progress. The aim of this paper is to study the decision of when to introduce a new product. To do so we employ the Impulse Control modeling approach that is perfectly suitable to take into account the disruptive changes caused by innovations. This also enables us to determine the length of the time interval that the firm uses a particular technology, when it is time to launch a new product generation, and how these decisions interact with the firm's capital accumulation behavior. In Kort (1989) a dynamic model of the firm is designed in which capital stock jumps upward at discrete points in time at which the firm invests. However, technological progress in not taken into account.

An example where a firm has to decide about investments in new generations of products is the LCD industry. With every new generation the size of the mother glass or substrate increases. As the LCD panels are cut out of the substrate, the substrate on the one hand determines which panel sizes can be produced and on the other hand how efficient each possible panel size can be produced. We have a process innovation, because a larger glass area provides a more efficient solution of the "cutting problem", and thus lower costs in the production process. A product innovation arises, because the larger area of the substrate makes it possible to produce larger screens. For a firm it is important to determine when it is optimal to introduce a new product. However, since the new product will decrease the demand of the old product, the moment of introduction is crucial.

Feichtinger et al. (2006) employs a vintage capital goods structure to study the effect of embodied technological progress on the investment behavior of the firm. They show that in the case that a firm has market power a negative anticipation effect occurs, i.e. when technological progress goes faster in the future, it is optimal for the firm to decrease investments in the current generation of capital goods. However, a direct implication of the vintage capital approach is that the firm adopts an infinite amount of different technologies. Of course, in practice a firm can adopt a new technology a limited number of times.

Grass et al. (2012) also combines technology adoption with capital accumulation, while taking into account technological progress. They find that investment jumps upward right at the moment that a new technology is adopted, and that the larger the firm the later the investment in a new technology takes place. Moreover, they find that when a firm has market power, the firm cuts down on investment before a new technology is adopted. Where Grass et al. (2012) limits itself to process innovation, we concentrate on studying product innovation. Grass et al. (2012) use a multi-stage optimal control approach where a firm adopts a new technology in each stage. Unlike Feichtinger et al. (2006), the number of technology adoptions is limited. However, the number of innovations is not determined by the model, but fixed exogenously instead. Unlike Feichtinger et al. (2006) and Grass et al. (2012), in this paper capital accumulation only occurs in lumps. Moreover, these lumps are determined by the model, i.e. the lumpy investments are endogenous. In Saglam (2011) a multi-stage optimal control model is studied where the number of technology adoptions are endogenous. However, unlike our paper, the model does not incorporate any (fixed) cost associated with the adoption and the considered firm has no market power. In Boucekkine et al. (2004) a two-stage optimal control model is considered, where only one adoption occurs, without adoption (fixed) cost. Both Boucekkine et al. (2004) and Saglam (2011) incorporate learning, were the firm raises productivity of a given technology over time due to learning and revenue is linear in the capital stock.

Our paper is mostly comparable with Grass et al. (2012). However, unlike Grass et al. (2012), we do not need to fix the number of technology adoptions beforehand and we do incorporate a (fixed) cost associated with this adoption. When dealing with product innovation, firms do not always have to scrap all capital goods. Sometimes measures are taken to allow new or updated parts to be fitted to old or outdated assemblies. As in Grass et al. (2012), we can model all situations in between the extreme cases where after every new investment the old capital goods are scrapped and the case where all the capital goods can be kept after adopting a new technology.

The method used to study firm behavior in this paper is Impulse Control. Impulse Control theory is a variant of optimal control theory where discontinuities (i.e. jumps) in the state variable are allowed. In Impulse Control the moments of these jumps as well as the sizes of the jumps are decision variables. Blaquière (1977a; 1977b; 1979; 1985) extends the standard theory on optimal control by deriving a Maximum Principle, the so-called Impulse Control Maximum Principle, that gives necessary and sufficient optimality conditions for solving such problems. Blaquière's Impulse Control analysis is based on the present value Hamiltonian form. In this paper we apply the Impulse Control theorem in the current value Hamiltonian framework as derived in Chahim et al. (2012).

One of the striking results is that the firm does not invest such that the marginal profit is zero, but instead marginal profit is negative. Furthermore, we obtain that the firm in the beginning of the planning period adopts new technologies faster as time proceeds, but after some moment in time later technologies are used for a longer time period. This behavior is different from Grass et al. (2012), who finds that the firm adopts new technologies faster as time proceeds for the whole planning period, but this also differs from the results found in Saglam (2011), who finds that later technologies are used during a longer time period. Our results are somehow a combination of both. Moreover, we find that if the time it takes to double the efficiency of technology is larger than the time it takes for the capital stock to depreciate, the firm undertakes an initial investment. Finally, we show that when demand decreases over time the firm invest less throughout the planning period and that the first investment is delayed.

This paper is organized as follows. In Section 2 we give the general setting and build up the Impulse Control model. Section 3 derives the necessary optimality conditions, whereas Section 4 gives a brief description of the algorithms present in the literature dealing with the Impulse Control Maximum Principle. In Section 5 we study the investment behavior of the firm, and in Section 6 we extend this analysis by adding decreasing demand, i.e. demand decreases over time due to competitors producing better products because of technological progress. Finally, in Section 7 we conclude and give some recommendations for future research.

2 The Model

Consider a firm that invests in lumps over time. Each time it invests it installs a production plant suitable to produce the new product. Due to product innovation the quality of the products, and thus also demand, increases over time. This implies that the later an investment takes place, the better products can be made due to these investments.

This is formalized as follows. A plant being installed at time τ will make products from which the price is given by the following inverse demand function:

$$p(t) = \theta(\tau) - q(t),$$

where $q(t)$ is the output at time t and $\theta(\tau) = 1 + b\tau$ is the state of technology that the firm adopts at time τ[1]. We further assume that technology within the firm does not change between two technology adoptions, i.e. $\dot{\theta} = 0$ for all $t \neq \tau$. At the moment the firm adopts a technology, the firms technology changes is given by

$$\theta(\tau_i^+) - \theta(\tau_i^-) = 1 + b\tau_i - \theta(\tau_i^-).$$

Hence, as in Feichtinger et al. (2006) and Grass et al. (2012) we impose that technological progress increases linearly over time, where b is a positive constant. In Saglam (2011) technology increases exponentially over time and in Boucekkine et al. (2004) there are only two different technologies available. We assume a simple production function in the sense that one capital good produces one unit of output. Denoting the stock of capital goods by $K(t)$, this gives

$$K(t) = q(t).$$

We impose that only the capital stock of the new plant is able to produce the new products, i.e. each plant has its own capital stock that produces the products with a quality associated with the timing of the investment in this plant. In this setting we can also model a situation where just $100\gamma\%$, where $\gamma \in [0, 1]$, of the capital stock is scrapped, while the remaining machines or tools can be reused for the new product. Hence, full scrapping corresponds to the case where $\gamma = 1$. This implies that old products, and thus also old capital goods, become worthless after the new plant is installed, implying that the old capital goods can be scrapped.

Denoting investment by $I(t)$, at the moment the firm invests (adopts a new technology) capital stock changes by

$$K(\tau^+) - K(\tau^-) = I(\tau) - \gamma K(\tau^-).$$

At time zero the capital stock is equal to zero, i.e.

$$K(0) = 0.$$

For each plant it holds that capital stock depreciates with rate δ, i.e.

$$\dot{K} = -\delta K.$$

Investing in a plant implies that the firm has to pay a fixed cost, i.e. part of the cost is independent of the plant size, and a variable cost that more than proportionally increases with the size of the plant. In particular, we assume that the investment cost is given by

$$C(I) = \begin{cases} C + \alpha I + \beta I^2 & \text{for } I > 0, \\ 0 & \text{for } I = 0. \end{cases}$$

[1]We assume that technology is continuously changing, i.e. $\theta(t) = 1 + bt$. However, the technology within the firm is the technology that the firm adopts at time τ.

This type of investment cost function, without the fixed cost, is common in the literature (e.g., among others, see Grass et al. (2012), Sethi and Thompson (2006) and Seierstad and Sydsæter (1987)), where besides the fixed cost, the linear term consist of acquisition cost, where the unit price is equal to α and the quadratic term represents the adjustment cost. In "ordinary" optimal control the investment cost function (i.e. the profit (utility or costs associated with the system) does not include a fixed cost, because this violates the continuity of the cost function with respect to its arguments, i.e. the control and the state variable.

Total discounted revenue is given by

$$\int_0^T e^{-rt} \left[\theta\left(t\right) - K\left(t\right)\right] K(t)dt, \tag{1}$$

where revenue is determined by output price times output. Since we have a finite time planning period, a salvage value has to be defined. This salvage value is equal to the value of the firm at the time horizon T. We assume that this value is give by

$$+e^{-rT}\frac{[\theta(\tau_N) - K(T^+)]K(T^+)}{r+\delta}. \tag{2}$$

The salvage value (2) is a lower bound of the discounted revenue stream of the firm after the planning period.

Total discounted investment cost are given by the sum of the cost of adopting a new technology, discounted at the time the adoption takes place. This results in

$$-\sum_{i=1}^{N} e^{-r\tau}\left(C + \alpha I\left(\tau_i\right) + \beta I\left(\tau_i\right)^2\right). \tag{3}$$

The above gives rise to the following impulse control model:

$$\begin{aligned}
&\max_{I,\tau_i,N} \int_0^T e^{-rt}\left[\theta\left(t\right) - K\left(t\right)\right] K(t)dt \\
&-\sum_{i=1}^{N} e^{-r\tau}\left(C + \alpha I\left(\tau_i\right) + \beta I\left(\tau_i\right)^2\right) \\
&+e^{-rT}\frac{[\theta(\tau_N) - K(T^+)]K(T^+)}{r+\delta}
\end{aligned} \tag{4}$$

subject to

$$\dot{K}\left(t\right) = -\delta K\left(t\right) \quad \text{for all} \quad t \neq \tau_1, ..., \tau_N, \tag{5}$$

$$\dot{\theta} = 0 \quad \text{for all} \quad t \neq \tau_1, ..., \tau_N, \tag{6}$$

$$K(\tau_i^+) - K(\tau_i^-) = I(\tau_i) - \gamma K(\tau_i^-) \quad \text{for all} \quad t = \tau_1, ..., \tau_N, \tag{7}$$

$$\theta(\tau_i^+) - \theta(\tau_i^-) = 1 + b\tau_i - \theta(\tau_i^-) \quad \text{for all} \quad t = \tau_1, ..., \tau_N, \tag{8}$$

$$K(0) = 0, \tag{9}$$

$$\theta\left(0\right) = 1. \tag{10}$$

This is an Impulse Control problem as described in Blaquière (1977a; 1977b; 1979; 1985). Note that this innovation model only contains an impulse control variable and no ordinary control variable. This approach differs from the multi-stage approach used in Grass et al. (2012), because here investment takes place in lumps and every investment goes along with a fixed cost. As in Grass et al. (2012) we can model all situations between the extreme cases where after every new investment the old capital goods are scrapped ($\gamma = 1$) and where all the capital can be kept ($\gamma = 0$) to produce the new product. Another benefit of the above model is that the number of technology changes are endogenous.

3 Necessary optimality conditions

We apply the impulse control maximum principle in current value formulation derived in Chahim et al. (2012). Other good references deriving the necessary optimality conditions for the Impulse Control problems are Blaquière (1977a; 1977b; 1979; 1985), Seierstad (1981) and Seierstad and Sydsæter (1987). We define the Hamiltonian Ham and the Impulse Hamiltonian $IHam$

$$\begin{aligned} Ham &= \left[\theta(t) - K(t)\right] K(t) - \lambda_1(t)\,\delta K(t), && (11)\\ IHam &= -C - \alpha I(\tau_i) - \beta I(\tau_i)^2 + \lambda_1\left(I(\tau_i) - \gamma K(\tau_i^-)\right) + \lambda_2\left(1 + b\tau_i - \theta(\tau_i^-)\right), && (12) \end{aligned}$$

and obtain the adjoint equations

$$\begin{aligned} \dot{\lambda}_1(t) &= (r+\delta)\,\lambda_1(t) - \theta(t) + 2K(t), && (13)\\ \dot{\lambda}_2(t) &= r\lambda_2(t) - K(t). && (14) \end{aligned}$$

The jump conditions are

$$\begin{aligned} -\alpha - 2\beta I(\tau_i) + \lambda_1\left(\tau_i^+\right) &= 0, && (15)\\ \lambda_1\left(\tau_i^+\right) - \lambda_1\left(\tau_i^-\right) &= \gamma\lambda_1\left(\tau_i^+\right), && (16)\\ \lambda_2\left(\tau_i^+\right) - \lambda_2\left(\tau_i^-\right) &= \lambda_2\left(\tau_i^+\right), && (17) \end{aligned}$$

from which we conclude that

$$\lambda_1\left(\tau_i^-\right) = (1-\gamma)\,\lambda_1\left(\tau_i^+\right),$$

which equals zero for $\gamma = 1$, and

$$\lambda_2\left(\tau_i^-\right) = 0.$$

The condition for determining the optimal switching time τ_i is

$$\begin{aligned} &Ham[\tau_i^+] - Ham[\tau_i^-] - \frac{\partial G\left(x\left(\tau_i^-\right), v_i, \lambda\left(\tau_i^+\right), \tau_i\right)}{\partial\tau} + rG\left(x\left(\tau_i^-\right), v_i, \lambda\left(\tau_i^+\right), \tau_i\right)\\ &-\lambda\left(\tau_i^+\right)\frac{\partial g[\tau_i^-]}{\partial\tau}\begin{cases} > 0 & \text{for } \tau_i^* = 0\\ = 0 & \text{for } \tau_i^* \in (0,T)\\ < 0 & \text{for } \tau_i^* = T. \end{cases} \end{aligned}$$

Using the above specification, we get

$$\begin{aligned} &\left[\theta\left(\tau_i^+\right) - K\left(\tau_i^+\right)\right] K\left(\tau_i^+\right) - \left[\theta\left(\tau_i^-\right) - K\left(\tau_i^-\right)\right] K\left(\tau_i^-\right)\\ &-\lambda_1\left(\tau_i^+\right)\delta K\left(\tau_i^+\right) + \lambda_1\left(\tau_i^-\right)\delta K\left(\tau_i^-\right) - rC - r\alpha I(\tau_i) - r\beta I(\tau_i)^2 - b\lambda_2\left(\tau_i^+\right)\\ &\begin{cases} > 0 & \text{for } \tau_i^* = 0\\ = 0 & \text{for } \tau_i^* \in (0,T)\\ < 0 & \text{for } \tau_i^* = T. \end{cases} && (18) \end{aligned}$$

The transversality conditions are

$$\lambda_1\left(T^{+}\right) = \frac{\theta(\tau_N) - 2K\left(T^{+}\right)}{r+\delta} \quad (19)$$

$$\lambda_2\left(T^{+}\right) = \frac{K(T^{+})}{r+\delta.} \quad (20)$$

At the non-jump points $t \neq \tau_1, ..., \tau_N$, it holds that $\lim_{I\to 0} \frac{\partial IHam}{\partial I} = \infty$ due to the fixed cost. Hence, the conditions for applying the Impulse Control Maximum Principle are met, see Section 2.3 of Chahim et al. (2012).

4 Algorithm

In the literature three different algorithms are derived based on the Impulse Control Maximum Principle (Blaquière (1977a; 1977b; 1979; 1985) and Chahim et al. (2012)). Luhmer (1986) derived a forward algorithm (starts at time 0) and Kort (1989, pp. 62-70) derived a backward algorithm (starts at final time horizon T). Luhmer (1986) starts at $t = 0$ and uses the costate variable, as input to initialize his algorithm. Kort (1989) implements a backward algorithm that starts at the time horizon, i.e. $t = T$, and initializes the algorithm using the values of the state variables. Finally, Grass and Chahim (2012) designs an algorithm that is a combination of continuation techniques and a multi-point Boundary Value Problem (BVP) to solve Impulse Control problems.

The problem described by (4)-(10) has two state variables, the stock of capital ($K(t)$) and technology ($\theta(t)$). The question is which algorithm is most suitable for this model. Applying the forward algorithm to problem (4)-(10) has a drawback. Namely, we have to guess the initial values for the two costate variables, λ_1 and λ_2. A wrong guess of the costate variables at the initial time results in a solution that does not satisfy the transversality conditions (19) and (20), which implies that the necessary optimality conditions are not satisfied. For the backward algorithm we start with choosing values for the state variables at time T. The resulting solution always satisfies the necessary optimality conditions, but here the problem is that the algorithm has to end up at the right $K(0)$. In other words, with the backward algorithm one can apply the right necessary conditions to the wrong problem. An example where the backward algorithm is applied successfully is Chahim et al. (2011). Moreover, in Chahim et al. (2011) clear upper and lower bounds have been derived for the state variable.

In addition, the backward algorithm has another drawback. When we apply it to the problem described by (4)-(10), starting at the time horizon and going back in time requires knowledge of the technology before the investment. In particular, we obtain from equation (18) that we need to know $\theta(\tau_N)^{+} = 1 + b\tau_N$ and $\theta(\tau_N^{-}) = \theta(\tau_{N-1}) = 1 + b\tau_{N-1}$. Hence, solving this equation for τ_N requires that we know τ_{N-1}. And with the backward algorithm, this predecessor is not known. We conclude that the backward algorithm is not suitable to solve our model.

The third algorithm described in the literature is an algorithm that considers the problem described by (4)-(10) as a multi-point Boundary Value Problem (BVP) and uses a continuation technique to solve it. The main idea behind the algorithm is as follows. To find the solution of the problem described by (4)-(10) we can apply a continuation strategy with respect to the time horizon T, i.e T is our continuation variable. The algorithm for this approach is described in Box 1. To initialize the algorithm, the problem is solved for $T = 0$. Assuming that a unique solution exists for $T = 0$, the initial conditions together with the transversality conditions combined with the necessary conditions results in a set of n equation with n unknowns. Given a solution for $T = 0$, T is increased (continued) during the continuation process whereas the conditions for possible jumps are monitored. If the conditions for a jump are satisfied, the BVP is adapted to this situation. With this new solution the continuation is pursued. Grass and Chahim (2012) describes this algorithm, based on Chahim et al. (2012), more extensively. Another paper on BVP and continuation is Grass (2012), but this one focusses on ordinary optimal control problems.

Define T as time horizon for the problem.
Define $\bar{T}$ to be a continuation variable.
Set $\bar{T} = 0$ and $\tau_l = 0$

Step 1: Find jump in $[\tau_l, \bar{T}]$ for:
- case 1: A jump occurs at the end, i.e. at $t = \bar{T}$, save as JumpSol
- case 2: No jump at the end, save as noJumpSol

Step 2: Start the continuation for $\bar{T} \in (\tau_l, T)$ with JumpSol until interior jump condition is satisfied, i.e. let $\bar{T}$ increase until (15)-(18) are satisfied for some $t = \tau$. Set $\tau_l = \tau$, save as JumpSol. Also continue the result of noJumpSol until $\bar{T} = \tau_l$, save as noJumpSol.
If $\bar{T} \geq T$ without satisfying interior jump conditions, stop.
- case 1: Objective of JumpSol higher than objective noJumpSol, add arc and go to step 1.
- case 2: Objective of JumpSol is lower than objective noJumpSol, go to step 3.

Step 3: Continue the solution of noJumpSol until the interior jump condition (15)-(18) is satisfied for $t = \tau \in (\tau_l, T)$. Set $\tau_l = \tau$ save as JumpSol, add arc, and go to step 1. If $\bar{T} \geq T$ without satisfying interior jump conditions, stop.

Box 1: Multi-point BVP and continuation algorithm

5 Endogenous lumpy investments

When a firm is dealing with market power, the output price decreases with the quantity that is produced. Since it holds in this model that with one unit of capital stock one unit of output is produced, we have that the output price decreases with the amount of capital. So during the time period between two investments the output price increases, since depreciation decreases capital stock. We consider no scrapping, partial scrapping and total scrapping, i.e. we consider $\gamma = 0$, $\gamma = 0.5$ and $\gamma = 1$. We provide a numerical analysis starting with the parameter values

$$b = \frac{1}{n} \log 2 = \frac{1}{2} \log 2, \quad \alpha = 0, \quad \beta = 0.2, \quad C = 2 \quad r = 0.04, \quad \delta = 0.2,$$

which we considers as the benchmark throughout this paper. As in Grass et al. (2012), we base our value for b on Moore's law[2], where the value for b is such that the efficiency of the technology doubles every n years where we put $n = 2$. The results of the first ten investments,

[2] Moore's law still holds, The Economist, July 14th 2012, Chipping in: A deal to keep Moores law alive.

are presented in Table 1 for $T = 100$. Table 6 of the Appendix presents all investments up until $T = 100$.

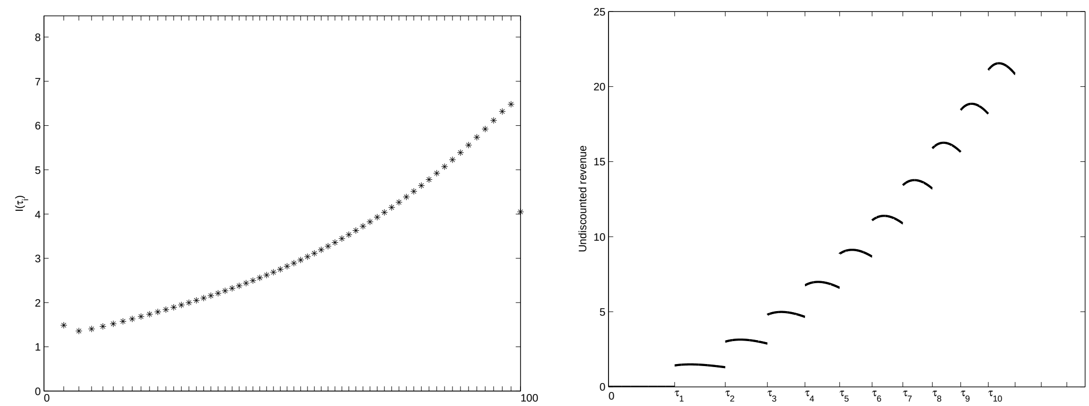
Ignoring the first and last investment, we see that the better the technology is, the larger the investment becomes. It seems as if the firm delays the first investment (compared to the others) to start production of a new good. In Figure 1(a) this is clearly seen (also see Figure 4(a) and Figure 6(a) in Appendix A). To understand what happens with the first investment we have to distinguish between $\gamma < 1$ and $\gamma = 1$. When $\gamma < 1$ capital growth is increased with each investment without fully scrapping the old capital stock. Because there is only limited scrapping, at an early stage the firm undertakes a relatively high investment to start production. A firm only undertakes this relatively high investment if there is limited scrapping, because the investments help to increase the capital stock in the future.

	$\gamma = 0$	$\gamma = 0.5$	$\gamma = 1$
$(\tau_i.I(\tau_i))$	4.1651 : 1.4877	4.1462 : 1.4682	3.8509 : 1.3689
	7.3464 : 1.3571	7.4147 : 1.7204	7.1308 : 1.9589
	10.0022 : 1.4032	10.1649 : 2.0101	9.9511 : 2.4614
	12.3693 : 1.4610	12.6433 : 2.2785	12.5559 : 2.9262
	14.5474 : 1.5188	14.9499 : 2.5312	15.0389 : 3.3716
	16.5895 : 1.5751	17.1370 : 2.7731	17.4476 : 3.8067
	18.5276 : 1.6299	19.2361 : 3.0070	19.8100 : 4.2365
	20.3835 : 1.6837	21.2682 : 3.2353	22.1437 : 4.6639
	22.1724 : 1.7365	23.2479 : 3.4594	24.4606 : 5.0910
	23.9056 : 1.7887	25.1861 : 3.6805	26.7688 : 5.5191
Revenue (discounted)	802.4809	790.1920	771.3955
Investment cost (discounted)	35.3109	67.8103	97.6050
Total profit (discounted)	767.1700	722.3817	673.7904

Table 1: First ten investments of Impulse Control solutions for γ. $T = 100$ and parameter values $r = 0.04$, $\delta = 0.2$, $b = \frac{1}{2}\log 2$, $\beta = 0.2$, $\alpha = 0$. $C = 2$, $K_0 = 0$ and $\theta(0) = 1$.

This behavior is clear to see in Figure 1(a). Drawing a line in the point of Figure 1(a) ignoring the first and last investment not only tells us that the first investment is relatively large, but also that the last investment is small. This last investment being small occurs due to the fact that the salvage value of the problem is (too) low, because it does not take into account technological improvement after time T.

Table 1 shows that the higher the scrapping percentage the larger the investments become. This makes sense because a firm that wants similar production has to invest extra to replace the scrapped parts. This scrapping increases the investment cost and at the same time, due to the quadratic term in the investment cost function, investing such that the same level of capital is reached as in the case of no scrapping, is too expensive. Hence, the optimal level of capital stock in the case of scrapping is lower than under no scrapping, which explains the lower revenue. Table 6 of Appendix A presents all investments up until $T = 100$ (Table 7, 8 and 9 present full results for $\gamma = 0$, $\frac{1}{2}$ and 1, respectively). It shows that a higher scrapping percentage decreases the number of investments during the planning period.



(a) Lumpy investments, $I(\tau_i)$

(b) Undiscounted revenue for the first ten investments

Figure 1: For $T = 100$ and parameter values $r = 0.04$, $\delta = 0.2$, $\gamma = 0$, $b = \frac{1}{2}\log 2$, $\beta = 0.2$, $\alpha = 0$, $C = 2$, $K_0 = 0$ and $\theta(0) = 1$

Another striking effect can be noticed when looking at Figure 1(b). We see that the firm invests in a new product such that marginal revenue is negative. In a "static" model (i.e. a model that does not depend on time) we know that the firms optimize profit and hence invest at the moment that marginal cost is equal to marginal revenue. Since we did not include any operation cost, we know that marginal cost is equal to zero. Hence, when marginal revenue is equal to zero, (i.e. $K(t) = \theta(\tau_i)/2$) investment would be optimal according to this rule. In our dynamic setting it is impossible to stay at the point where marginal revenue is equal to zero, due to depreciation. In Table 2 we show the results for a case where we have no depreciation. We see that indeed the investments are such that the level of capital is set to $K(t) = \theta(\tau_i)/2$. In the case that we have depreciation, the firm overinvests, i.e., invests such that marginal revenue is negative. Then up until the next investment, marginal revenue increases, becomes zero after some time, and then turns positive.

τ_i	$\theta(\tau_i^+)$	$K(\tau_i^+)$	$\frac{\theta}{K}$
19.6234	7.8009	3.8574	2.0224
34.5329	12.9682	6.4650	2.0059
50.7184	18.5777	9.2706	2.0039
70.6244	25.4766	12.7165	2.0034
99.7453	35.5691	17.7443	2.0045

Table 2: Technology level and capital for $T = 100$ and parameter values $\gamma = 0$, $r = 0.04$, $\delta = 0$, $b = \frac{1}{2}\log 2$, $\beta = 0.2$, $\alpha = 0$, $C = 2$, $K_0 = 0$ and $\theta(0) = 1$.

In Figure 2 we have plotted the length of the time interval between two investments. We see that in the beginning of the planning period the firm adopts new technologies faster as time proceeds and after some moment it uses later technologies for a longer time period. This behavior is different from Grass et al. (2012), who finds that the firm adopt new technologies faster as time proceeds for the whole planning period, but this also differs from the results found in Saglam (2011), who finds that later technologies used during a longer time period. Our results are somehow a combination of both. An explanation for this could be that the firm in the beginning of the planning period does not invest much since productivity is low. After some time technological progress is such that each investment is more profitable, which makes that the corresponding capital goods are used for a longer time. For this reason the time between investments increases. Also for higher T a similar effect is found.

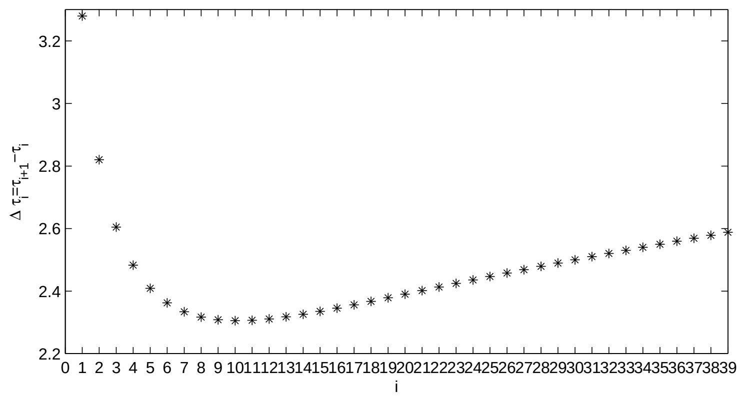


Figure 2: The length between two investments for $T = 100$ and parameter values $r = 0.04$, $\delta = 0.2$, $\gamma = 0$, $b = \frac{1}{2}\log 2$, $\beta = 0.2$, $\alpha = 0$, $C = 2$, $K_0 = 0$ and $\theta(0) = 1$

5.1 Sensitivity analysis with respect to the rate of technology change

Here we study how the rate of technological progress affects the investment behavior of a firm. Remember that we have assumed, using Moore's law, that efficiency of technology doubles every n years, setting $n = 2$ for our benchmark case. Table 3 shows the first ten investments for different values of the technology rate b. For all investments up until $T = 100$ see Table 10 of Appendix A (or Table 11-15 for each level of technology change separately). When $n > 5$ an investment takes place at $t = 0$. The explanation behind this is that for $n > 5$ we have, under Moore's law, that it takes more than five years for the efficiency of a technology to double. Since we have a depreciation rate of 20%, this means that the firm's capital stock is (almost) depreciated before the efficiency of technology doubles. So the firm has no incentive to wait and invests at $t = 0$.

	$b = \frac{1}{3}\log 2$	$b = \frac{1}{4}\log 2$	$b = \frac{1}{5}\log 2$	$b = \frac{1}{6}\log 2$	$b = \frac{1}{10}\log 2$
$(\tau_i.I(\tau_i))$	4.6759 : 1.3116	5.1658 : 1.2381	5.5832 : 1.1914	0 : 0.7418	0 : 0.7752
	8.6561 : 1.5392	9.7814 : 1.4539	10.7534 : 1.3980	7.7656 : 1.2080	9.7219 : 1.1432
	1.9662 : 1.7807	13.5911 : 1.6692	14.9977 : 1.5949	13.0448 : 1.4152	16.3705 : 1.3132
	14.9229 : 1.9995	16.9755 : 1.8614	18.7535 : 1.7685	17.4932 : 1.5907	21.9534 : 1.4524
	17.6530 : 2.2025	20.0857 : 2.0378	22.1932 : 1.9266	21.4683 : 1.7459	26.9204 : 1.5730
	20.2231 : 2.3943	23.0005 : 2.2031	25.4066 : 2.0736	25.1251 : 1.8873	31.4676 : 1.6810
	22.6732 : 2.5779	25.7678 : 2.3601	28.4478 : 2.2125	28.5485 : 2.0188	35.7025 : 1.7797
	25.0300 : 2.7553	28.4191 : 2.5108	31.3530 : 2.3450	31.7914 : 2.1429	39.6921 : 1.8712
	27.3121 : 2.9280	30.9766 : 2.6566	34.1472 : 2.4725	34.8894 : 2.2610	43.4813 : 1.9569
	29.5335 : 3.0970	33.4569 : 2.7986	36.8494 : 2.5960	37.8681 : 2.3745	47.1023 : 2.0376
Revenue (discounted)	371.5616	220.0775	148.0959	108.6965	47.4170
Investment cost (discounted)	39.2258	27.6829	21.7123	19.5772	12.9673
Total profit (discounted)	332.3358	192.3946	126.3837	89.1193	34.4497

Table 3: First ten investments of Impulse Control solutions for b, $T = 100$ and parameter values $\gamma = 0.5$, $r = 0.04$. $\delta = 0.2$, $\beta = 0.2$, $\alpha = 0$. $C = 2$, $K_0 = 0$ and $\theta(0) = 1$.

5.2 Sensitivity analysis with respect to the fixed cost

One of the main differences between Grass et al. (2012), Boucekkine et al. (2004) and Saglam (2011) is that they do not incorporate any (fixed) cost) and this paper assumes that a fixed cost is included for each investment. Here we study how increasing these fixed cost affects the investment behavior of a firm. Table 4 shows the first ten investments for each size of fixed cost. For all investments up until $T = 100$ see Table 16 of Appendix A (or Table 17-20 for each for each size of fixed cost separately). It is easily seen, that if we increase the fixed cost, the first investment is delayed and at the same time the time period between two investments increases. Hence, the number of investments decreases if the fixed cost increase. Comparing the results more carefully, we see that the size of the lumpy investments (i.e. jumps) increases, when C increases.

	$C = 4$	$C = 8$	$C = 16$	$C = 32$
$(\tau_i.I(\tau_i))$	5.7915 : 1.8832	8.0844 : 2.4856	11.1517 : 3.3199	15.2866 : 4.4754
	9.6593 : 2.2099	12.7147 : 2.9206	16.6712 : 3.8947	21.8148 : 5.2241
	12.8816 : 2.5607	16.5386 : 3.3546	21.1933 : 4.4297	27.1293 : 5.8789
	15.7638 : 2.8797	19.9372 : 3.7422	25.1901 : 4.8993	31.8052 : 6.4443
	18.4283 : 3.1763	23.0621 : 4.0984	28.8471 : 5.3256	36.0657 : 6.9513
	20.9394 : 3.4571	25.9923 : 4.4325	32.2606 : 5.7215	40.0266 : 7.4169
	23.3358 : 3.7265	28.7755 : 4.7502	35.4889 : 6.0947	43.7577 : 7.8515
	25.6433 : 3.9871	31.4435 : 5.0556	38.5705 : 6.4506	47.3050 : 8.2618
	27.8799 : 4.2412	34.0186 : 5.3513	41.5327 : 6.7926	50.7012 : 8.6525
	30.0590 : 4.4903	36.5173 : 5.6394	44.3957 : 7.1237	53.9701 : 9.0270
Revenue (discounted)	780.7835	769.1875	747.0746	712.6433
Investment cost (discounted)	79.5936	96.8939	120.5584	150.9987
Total profit (discounted)	701.1899	672.2936	626.5162	561.6447

Table 4: Impulse Control solutions for C, $T = 100$ and parameter values $\gamma = 0.5$, $r = 0.04$, $\delta = 0.2$, $b = \frac{1}{2}\log 2$, $\beta = 0.2$, $\alpha = 0$, $K_0 = 0$ and $\theta(0) = 1$.

6 Lumpy investments under decreasing demand

In this section we consider the case where the demand for an existing product decreases over time. A main reason could be that the competitors' products become better due to their product innovations.

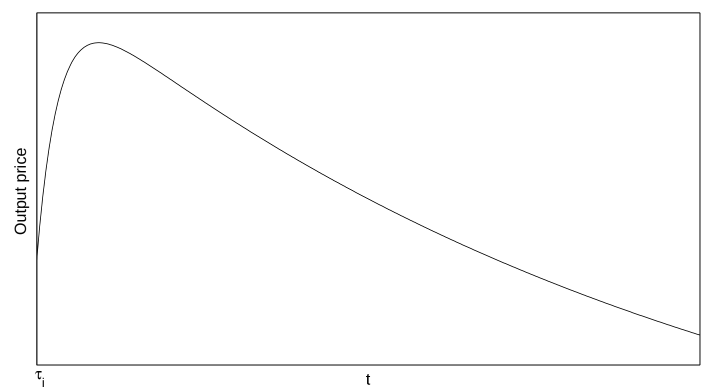


Figure 3: output price as a function in time for $\delta > \eta$.

We incorporate decreasing demand by setting $\dot{\theta} = -\eta t$, where η is some positive constant. Since it is reasonable to assume $\delta > \eta > 0$[3] the output price after investment is first increasing and then decreasing, see Figure 3. Hence, if a firm invests capital stock depreciates and the output price increases, and after some time this output price is decreasing due to this decreasing demand. Then the model becomes

[3]Since we are dealing with product innovation and assume a depreciation rate of 20% it is unlikely that demand decreases by more than (or equal to) 20% and hence we do not consider the case that $\eta \geq \delta > 0$.

$$\max_{i,I,\tau_i,N} \int_0^T e^{-rt} \left[\theta(t) - K(t)\right] K(t) dt$$
$$- \sum_{i=1}^{N} e^{-rt} \left(C + \alpha I(\tau_i) + \beta I(\tau_i)^2 \right)$$
$$+ e^{-rT} \frac{[\theta(T^+) - K(T^+)]K(T^+)}{r + \delta + \eta}, \tag{21}$$

subject to

$$\begin{aligned}
\dot{K}(t) &= -\delta K(t) \quad \text{for all} \quad t \neq \tau_1, ..., \tau_N, && (22) \\
\dot{\theta}(t) &= -\eta \theta(t) \quad \text{for all} \quad t \neq \tau_1, ..., \tau_N, && (23) \\
K(\tau_i^+) - K(\tau_i^-) &= I(\tau_i) - \gamma K(\tau_i^-) \quad \text{for all} \quad t = \tau_1, ..., \tau_N, && (24) \\
\theta(\tau_i^+) - \theta(\tau_i^-) &= 1 + b\tau_i - \theta(\tau_i^-) \quad \text{for all} \quad t = \tau_1, ..., \tau_N, && (25) \\
K(0) &= 0, && (26) \\
\theta(0) &= 1. && (27)
\end{aligned}$$

Remember that in Section 5 the output price was decreasing in capital. Hence, due to depreciation the output price is increasing in the time period between two investments. Since we are considering product innovation, it makes more sense that demand of a give product during the time period decreases. This is because over time new products are invented by other firms, which reduce demand of the current product. This demand decrease has a negative effect on output price and hence the firm has even a greater incentive to invest in a new technology.

Looking at the results of Table 5 and Table 21 (or Table 22-24 for each decay rate of the demand separately) we can see that a change in the decrease of demand directly affects the investment behavior. It is clear to see, that if we increase η the first investment is delayed (compared to a smaller η) and at the same time the time period between two investments also increases. Hence, the number of investments decreases if the decay rate of the demand increases. This makes sense, since less demand makes investing less attractive. This results in a lower investment cost for higher η. Moreover, the larger η the lower the output price (compared to a lower η) and hence the lower the revenue.

	$\eta = 0.01$	$\eta = 0.02$	$\eta = 0.03$
$(\tau_i.I(\tau_i))$	5.2730 : 1.7250	6.3504 : 1.9594	7.5126 : 2.2042
	8.9696 : 2.0366	10.4003 : 2.3175	11.902 : 2.6060
	12.0850 : 2.3821	13.8098 : 2.7062	15.5932 : 3.0359
	14.9011 : 2.7029	16.8941 : 3.0676	18.9345 : 3.4366
	17.5308 : 3.0067	19.7779 : 3.4110	22.0629 : 3.8188
	20.0327 : 3.2991	22.5261 : 3.7427	25.0493 : 4.1897
	22.4425 : 3.5837	25.1779 : 4.0670	27.9368 : 4.5542
	24.7835 : 3.8631	27.7594 : 4.3869	30.7539 : 4.9156
	27.0723 : 4.1393	30.2889 : 4.7047	33.5212 : 5.2765
	29.3212 : 4.4136	32.7803 : 5.0219	36.2541 : 5.6390
Revenue (discounted)	762.5966	733.2291	701.2148
Investment cost (discounted)	61.1145	56.6083	52.6074
Total profit (discounted)	701.4821	676.6208	648.6074

Table 5: First ten investments of Impulse Control solutions for η, $T = 100$ and parameter values $\gamma = 0.5$, $r = 0.04$, $\delta = 0.2$, $b = \frac{1}{2}\log 2$, $\beta = 0.2$, $\alpha = 0$, $C = 2$, $K_0 = 0$ and $\theta(0) = 1$.

7 Conclusions

This paper employs an Impulse Control modeling approach that is perfectly suitable to take into account the disruptive changes caused by innovations. We describe and implement an algorithm based on current value necessary optimality conditions. The necessary conditions are solved using a multi-point Boundary Value Problem (BVP) combined with some continuation techniques.

From an economic point of view we have derived some guidelines for lumpy investments in new technology:

- A striking result is that the firm does not invest such that marginal profit is zero, but instead marginal profit is negative. Indeed, due to depreciation capital stock decreases in between two investments, implying that marginal profit goes up there due to the decreasing returns to scale assumption. The implication is that during such an interval first marginal profit is negative, but then after a while it turns positive and this stays that way until it is time for the next investment.

- We find that investments are larger and the time between investments is larger when more of the old capital stock needs to be scrapped. If a change in technology permits the firm to keep, update and reuse part of its capital stock, the investments are smaller.

- A nontrivial result is the optimal timing of investments. We see that the firm in the beginning of the planning period adopts new technologies faster as time proceeds, but later on the opposite happens. Moreover, we obtain that the firm's investments increase when the technology produces more profitable products.

- Numerical experiments show that if the time it takes to double the efficiency of a technology is larger than the time it takes for the capital stock to depreciate to half of its original level, the firm undertakes an initial investment.

- Further sensitivity results were provided for a scenario of decreasing demand. We find that when demand decreases over time and when fixed investment cost is higher, then the firm invests less throughout the planning period, the time between two investments increases and the first investment is delayed.

Interesting directions for further work would be to consider running cost in the model or to introduce a learning effect. Another possible extension would be to let the scrapping percentage depend on time.

Appendix A: tables and figures for all cases

	$\gamma = 0$	$\gamma = 0.5$	$\gamma = 1$
$(\tau_i . I(\tau_i))$	4.1651 : 1.4877	4.1462 : 1.4682	3.8509 : 1.3689
	7.3464 : 1.3571	7.4147 : 1.7204	7.1308 : 1.9589
	10.0022 : 1.4032	10.1649 : 2.0101	9.9511 : 2.4614
	12.3693 : 1.4610	12.6433 : 2.2785	12.5559 : 2.9262
	14.5474 : 1.5188	14.9499 : 2.5312	15.0389 : 3.3716
	16.5895 : 1.5751	17.1370 : 2.7731	17.4476 : 3.8067
	18.5276 : 1.6299	19.2361 : 3.0070	19.8100 : 4.2365
	20.3835 : 1.6837	21.2682 : 3.2353	22.1437 : 4.6639
	22.1724 : 1.7365	23.2479 : 3.4594	24.4606 : 5.0910
	23.9056 : 1.7887	25.1861 : 3.6805	26.7688 : 5.5191
	25.5920 : 1.8407	27.0909 : 3.8994	29.0742 : 5.9490
	27.2385 : 1.8924	28.9689 : 4.1168	31.3809 : 6.3813
	28.8508 : 1.9443	30.8252 : 4.3333	33.6920 : 6.8164
	30.4336 : 1.9964	32.6640 : 4.5493	36.0096 : 7.2545
	31.9908 : 2.0488	34.4889 : 4.7652	38.3355 : 7.6957
	33.5258 : 2.1018	36.3027 : 4.9814	40.6707 : 8.1403
	35.0416 : 2.1554	38.1081 : 5.1982	43.0162 : 8.5881
	36.5406 : 2.2098	39.9072 : 5.4157	45.3723 : 9.0393
	38.0252 : 2.2651	41.7019 : 5.6343	47.7396 : 9.4937
	39.4975 : 2.3214	43.4940 : 5.8541	50.1182 : 9.9514
	40.9591 : 2.3788	45.2849 : 6.0753	52.5083 : 10.4123
	42.4119 : 2.4374	47.0759 : 6.2982	54.9099 : 10.8763
	43.8573 : 2.4973	48.8684 : 6.5229	57.3230 : 11.3435
	45.2968 : 2.5586	50.6635 : 6.7495	59.7474 : 11.8136
	46.7317 : 2.6214	52.4621 : 6.9783	62.1832 : 12.2867
	48.1632 : 2.6859	54.2654 : 7.2094	64.6300 : 12.7627
	49.5925 : 2.7520	56.0743 : 7.4431	67.0879 : 13.2415
	51.0207 : 2.8200	57.8895 : 7.6793	69.5566 : 13.7231
	52.4488 : 2.8900	59.7121 : 7.9184	72.0359 : 14.2075
	53.8779 : 2.9620	61.5428 : 8.1606	74.5258 : 14.6945
	55.3089 : 3.0362	63.3824 : 8.4059	77.0260 : 15.1843
	56.7427 : 3.1127	65.2318 : 8.6546	79.5364 : 15.6766
	58.1804 : 3.1917	67.0917 : 8.9070	82.0568 : 16.1716
	59.6228 : 3.2732	68.9629 : 9.1631	84.5872 : 16.6692
	61.0707 : 3.3574	70.8463 : 9.4233	87.1274 : 17.1693
	62.5251 : 3.4445	72.7426 : 9.6878	89.6772 : 17.6720
	63.9868 : 3.5347	74.6526 : 9.9569	92.2367 : 18.1774
	65.4567 : 3.6279	76.5773 : 10.2307	94.8058 : 18.6853
	66.9357 : 3.7246	78.5174 : 10.5097	97.3844 : 19.1959
	68.4245 : 3.8248	80.4738 : 10.7940	99.9725 : 17.0969
	69.9242 : 3.9287	82.4476 : 11.0841	
	71.4355 : 4.0366	84.4396 : 11.3803	
	72.9593 : 4.1486	86.4508 : 11.6829	
	74.4967 : 4.2650	88.4824 : 11.9925	
	76.0484 : 4.3860	90.5354 : 12.3093	
	77.6154 : 4.5120	92.6110 : 12.6340	
	79.1987 : 4.6431	94.7105 : 12.9668	
	80.7994 : 4.7798	96.8355 : 13.2846	
	82.4183 : 4.9222	99.0358 : 10.8535	
	84.0566 : 5.0708		
	85.7154 : 5.2260		
	87.3959 : 5.3881		
	89.0991 : 5.5576		
	90.8264 : 5.7349		
	92.5790 : 5.9206		
	94.3584 : 6.1152		
	96.1659 : 6.3181		
	98.0055 : 6.4796		
	99.9896 : 4.0490		
Revenue (discounted)	802.4809	790.1920	771.3955
Investment cost (discounted)	35.3109	67.8103	97.6050
Total profit (discounted)	767.1700	722.3817	673.7904

Table 6: Impulse Control solutions for γ, $T = 100$ and parameter values $r = 0.04$, $\delta = 0.2$, $b = \frac{1}{2}\log 2$, $\beta = 0.2$, $\alpha = 0$, $C = 2$, $K_0 = 0$ and $\theta(0) = 1$.

τ_i	$I(\tau_i)$	$K(\tau_i^-)$	$K(\tau_i^+)$	$\theta(\tau_i^+)$
4.1651	1.4877	0	1.4877	2.4435
7.3464	1.3571	0.7874	2.1445	3.5461
10.0022	1.4032	1.2608	2.6640	4.4665
12.3693	1.4610	1.6593	3.1204	5.2869
14.5474	1.5188	2.0184	3.5372	6.0418
16.5895	1.5751	2.3512	3.9263	6.7495
18.5276	1.6299	2.6646	4.2946	7.4212
20.3835	1.6837	2.9629	4.6466	8.0644
22.1724	1.7365	3.2490	4.9855	8.6844
23.9056	1.7887	3.5250	5.3138	9.2851
25.5920	1.8407	3.7925	5.6332	9.8695
27.2385	1.8924	4.0526	5.9451	10.4402
28.8508	1.9443	4.3064	6.2507	10.9989
30.4336	1.9964	4.5546	6.5510	11.5475
31.9908	2.0488	4.7979	6.8467	12.0872
33.5258	2.1018	5.0368	7.1386	12.6192
35.0416	2.1554	5.2718	7.4272	13.1445
36.5406	2.2098	5.5033	7.7131	13.6640
38.0252	2.2651	5.7316	7.9968	14.1785
39.4975	2.3214	5.9571	8.2786	14.6888
40.9591	2.3788	6.1801	8.5589	15.1954
42.4119	2.4374	6.4007	8.8381	15.6988
43.8573	2.4973	6.6193	9.1166	16.1998
45.2968	2.5586	6.8359	9.3945	16.6987
46.7317	2.6214	7.0509	9.6723	17.1960
48.1632	2.6859	7.2642	9.9501	17.6921
49.5925	2.7520	7.4762	10.2282	18.1875
51.0207	2.8200	7.6869	10.5069	18.6824
52.4488	2.8900	7.8964	10.7864	19.1774
53.8779	2.9620	8.1049	11.0670	19.6726
55.3089	3.0362	8.3125	11.3488	20.1686
56.7427	3.1127	8.5193	11.6320	20.6655
58.1804	3.1917	8.7253	11.9170	21.1638
59.6228	3.2732	8.9307	12.2039	21.6637
61.0707	3.3574	9.1355	12.4929	22.1655
62.5251	3.4445	9.3398	12.7843	22.6696
63.9868	3.5347	9.5437	13.0783	23.1761
65.4567	3.6279	9.7472	13.3751	23.6856
66.9357	3.7246	9.9503	13.6749	24.1981
68.4245	3.8248	10.1532	13.9780	24.7141
69.9242	3.9287	10.3559	14.2846	25.2339
71.4355	4.0366	10.5584	14.5950	25.7577
72.9593	4.1486	10.7607	14.9093	26.2858
74.4967	4.2650	10.9630	15.2279	26.8186
76.0484	4.3860	11.1651	15.5511	27.3564
77.6154	4.5120	11.3671	15.8791	27.8994
79.1987	4.6431	11.5691	16.2122	28.4482
80.7994	4.7798	11.7710	16.5508	29.0029
82.4183	4.9222	11.9729	16.8951	29.5640
84.0566	5.0708	12.1747	17.2455	30.1318
85.7154	5.2260	12.3764	17.6023	30.7067
87.3959	5.3881	12.5780	17.9660	31.2891
89.0991	5.5576	12.7794	18.3370	31.8794
90.8264	5.7349	12.9807	18.7156	32.4780
92.5790	5.9206	13.1817	19.1023	33.0854
94.3584	6.1152	13.3824	19.4975	33.7021
96.1659	6.3181	13.5825	19.9006	34.3286
98.0055	6.4796	13.7746	20.2542	34.9661
99.9896	4.0490	13.6201	17.6691	35.6538
Revenue (discounted)			790.1920	
Investment cost (discounted)			67.8103	
Total profit (discounted)			722.3817	

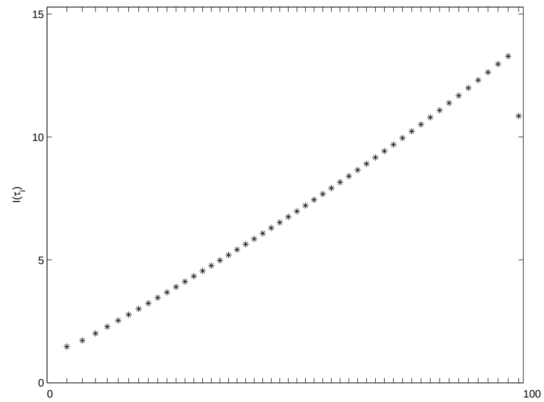
Table 7: Impulse Control solutions for $\gamma = 0$, $T = 100$ and parameter values $r = 0.04$, $\delta = 0.2$, $b = \frac{1}{2}\log 2$, $\beta = 0.2$, $\alpha = 0$, $C = 2$, $K_0 = 0$ and $\theta(0) = 1$.

τ_i	$I(\tau_i)$	$K(\tau_i^-)$	$K(\tau_i^+)$	$\theta(\tau_i^+)$
4.1462	1.4682	0.0000	1.4682	2.4370
7.4147	1.7204	0.7637	2.1022	3.5697
10.1649	2.0101	1.2128	2.6165	4.5229
12.6433	2.2785	1.5938	3.0754	5.3818
14.9499	2.5312	1.9389	3.5007	6.1812
17.1370	2.7731	2.2604	3.9033	6.9392
19.2361	3.0070	2.5651	4.2896	7.6667
21.2682	3.2353	2.8570	4.6638	8.3710
23.2479	3.4594	3.1390	5.0289	9.0571
25.1861	3.6805	3.4129	5.3869	9.7288
27.0909	3.8994	3.6803	5.7395	10.3890
28.9689	4.1168	3.9424	6.0880	11.0399
30.8252	4.3333	4.1999	6.4332	11.6832
32.6640	4.5493	4.4536	6.7761	12.3205
34.4889	4.7652	4.7041	7.1173	12.9529
36.3027	4.9814	4.9518	7.4574	13.5816
38.1081	5.1982	5.1972	7.7968	14.2073
39.9072	5.4157	5.4406	8.1360	14.8308
41.7019	5.6343	5.6823	8.4754	15.4528
43.4940	5.8541	5.9225	8.8153	16.0739
45.2849	6.0753	6.1615	9.1561	16.6945
47.0759	6.2982	6.3994	9.4979	17.3153
48.8684	6.5229	6.6364	9.8411	17.9365
50.6635	6.7495	6.8727	10.1859	18.5586
52.4621	6.9783	7.1083	10.5325	19.1820
54.2654	7.2094	7.3434	10.8812	19.8070
56.0743	7.4431	7.5781	11.2321	20.4339
57.8895	7.6793	7.8125	11.5856	21.0630
59.7121	7.9184	8.0466	11.9417	21.6946
61.5428	8.1606	8.2805	12.3008	22.3291
63.3824	8.4059	8.5142	12.6630	22.9667
65.2318	8.6546	8.7479	13.0286	23.6076
67.0917	8.9070	8.9815	13.3977	24.2522
68.9629	9.1631	9.2151	13.7707	24.9007
70.8463	9.4233	9.4486	14.1477	25.5535
72.7426	9.6878	9.6822	14.5290	26.2107
74.6526	9.9569	9.9159	14.9148	26.8726
76.5773	10.2307	10.1495	15.3055	27.5397
78.5174	10.5097	10.3832	15.7013	28.2120
80.4738	10.7940	10.6169	16.1025	28.8901
82.4476	11.0841	10.8506	16.5094	29.5741
84.4396	11.3803	11.0843	16.9224	30.2645
86.4508	11.6829	11.3179	17.3419	30.9616
88.4824	11.9925	11.5514	17.7682	31.6657
90.5354	12.3093	11.7848	18.2017	32.3772
92.6110	12.6340	12.0179	18.6429	33.0965
94.7105	12.9668	12.2506	19.0921	33.8241
96.8355	13.2846	12.4817	19.5255	34.5606
99.0358	10.8535	12.5743	17.1406	35.3232
Revenue (discounted)			802.4809	
Investment cost (discounted)			35.3109	
Total profit (discounted)			767.1700	

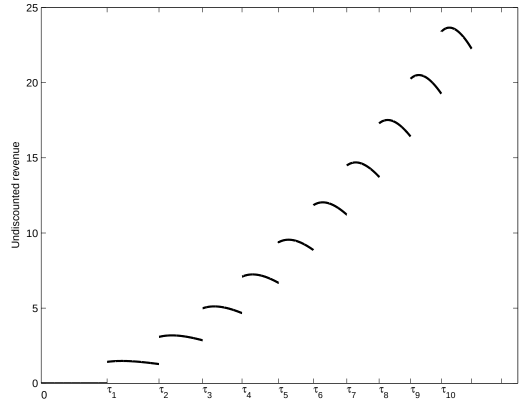
Table 8: Impulse Control solutions for $\gamma = 0.5$, $T = 100$ and parameter values $r = 0.04$, $\delta = 0.2$, $b = \frac{1}{2}\log 2$, $\beta = 0.2$, $\alpha = 0$, $C = 2$, $K_0 = 0$ and $\theta(0) = 1$.

τ_i	$I(\tau_i)$	$K(\tau_i^-)$	$K(\tau_i^+)$	$\theta(\tau_i^+)$
3.8509	1.3689	0	1.3689	2.3346
7.1308	1.9589	0.7104	1.9589	3.4713
9.9511	2.4614	1.1144	2.4614	4.4488
12.5559	2.9262	1.4619	2.9262	5.3516
15.0389	3.3716	1.7809	3.3716	6.2121
17.4476	3.8067	2.0827	3.8067	7.0469
19.8100	4.2365	2.3733	4.2365	7.8656
22.1437	4.6639	2.6564	4.6639	8.6744
24.4606	5.0910	2.9343	5.0910	9.4774
26.7688	5.5191	3.2086	5.5191	10.2774
29.0742	5.9490	3.4804	5.9490	11.0763
31.3809	6.3813	3.7505	6.3813	11.8758
33.6920	6.8164	4.0195	6.8164	12.6767
36.0096	7.2545	4.2879	7.2545	13.4800
38.3355	7.6957	4.5560	7.6957	14.2861
40.6707	8.1403	4.8241	8.1403	15.0954
43.0162	8.5881	5.0924	8.5881	15.9083
45.3723	9.0393	5.3610	9.0393	16.7248
47.7396	9.4937	5.6301	9.4937	17.5453
50.1182	9.9514	5.8997	9.9514	18.3697
52.5083	10.4123	6.1700	10.4123	19.1980
54.9099	10.8763	6.4409	10.8763	20.0303
57.3230	11.3435	6.7125	11.3435	20.8666
59.7474	11.8136	6.9849	11.8136	21.7069
62.1832	12.2867	7.2580	12.2867	22.5510
64.6300	12.7627	7.5319	12.7627	23.3991
67.0879	13.2415	7.8065	13.2415	24.2509
69.5566	13.7231	8.0818	13.7231	25.1065
72.0359	14.2075	8.3579	14.2075	25.9658
74.5258	14.6945	8.6348	14.6945	26.8287
77.0260	15.1843	8.9123	15.1843	27.6952
79.5364	15.6766	9.1906	15.6766	28.5652
82.0568	16.1716	9.4696	16.1716	29.4387
84.5872	16.6692	9.7492	16.6692	30.3157
87.1274	17.1693	10.0294	17.1693	31.1960
89.6772	17.6720	10.3103	17.6720	32.0798
92.2367	18.1774	10.5918	18.1774	32.9668
94.8058	18.6853	10.8739	18.6853	33.8572
97.3844	19.1959	11.1565	19.1959	34.7509
99.9725	17.0969	11.4396	17.0969	35.6478
Revenue (discounted)			771.3955	
Investment cost (discounted)			97.6050	
Total profit (discounted)			673.7904	

Table 9: Impulse Control solutions for $\gamma = 1$, $T = 100$ and parameter values $r = 0.04$, $\delta = 0.2$, $b = \frac{1}{2}\log 2$, $\beta = 0.2$, $\alpha = 0$, $C = 2$, $K_0 = 0$ and $\theta(0) = 1$.



(a) Lumpy investments, $I(\tau_i)$



(b) Undiscounted revenue for the first ten investments

Figure 4: For $T = 100$ and parameter values $r = 0.04$, $\delta = 0.05$, $\gamma = 0.5$, $b = \frac{1}{2}\log 2$, $\beta = 0.2$, $\alpha = 0$, $C = 2$, $K_0 = 0$ and $\theta(0) = 1$

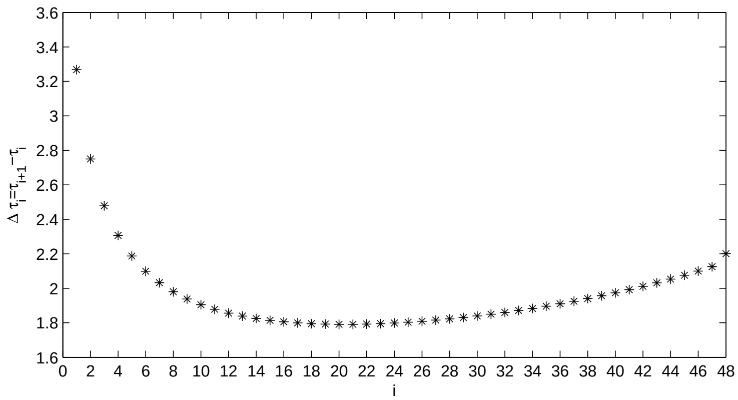
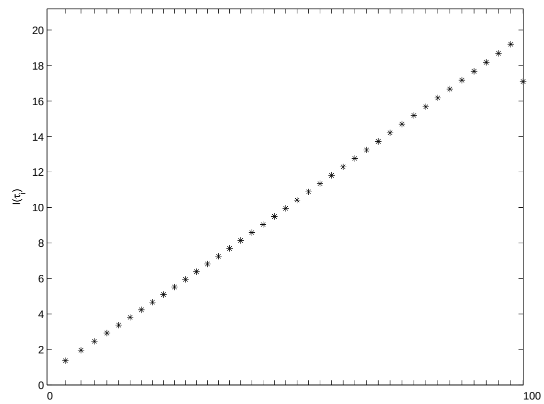
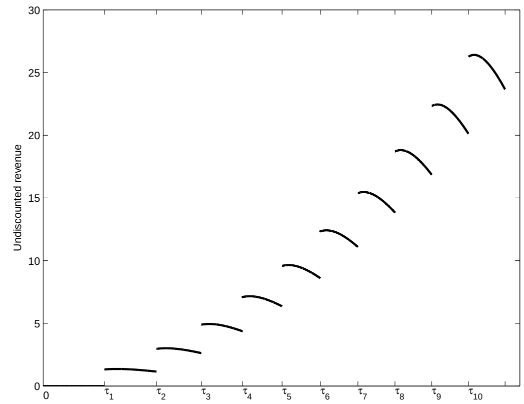


Figure 5: For $T = 100$ and parameter values $r = 0.04$, $\delta = 0.2$, $\gamma = 0.5$, $b = \frac{1}{2}\log 2$, $\beta = 0.2$, $\alpha = 0$, $C = 2$, $K_0 = 0$ and $\theta(0) = 1$



(a) Lumpy investments, $I(\tau_i)$



(b) Undiscounted revenue for the first ten investments

Figure 6: For $T = 100$ and parameter values $r = 0.04$, $\delta = 0.05$, $\gamma = 1$, $b = \frac{1}{2}\log 2$, $\beta = 0.2$, $\alpha = 0$, $C = 2$, $K_0 = 0$ and $\theta(0) = 1$

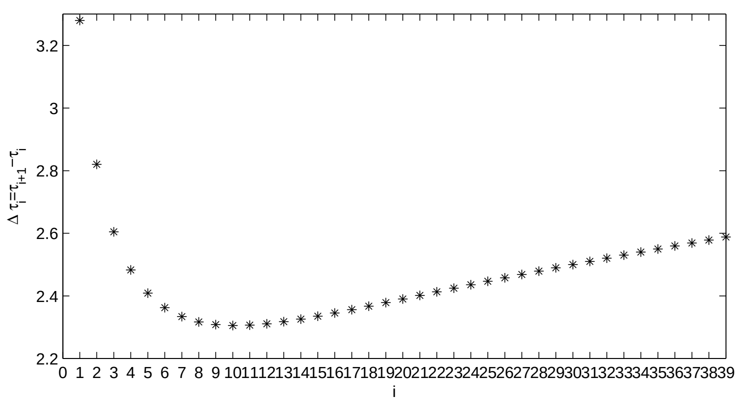


Figure 7: For $T = 100$ and parameter values $r = 0.04$, $\delta = 0.2$, $\gamma = 1$, $b = \frac{1}{2}\log 2$, $\beta = 0.2$, $\alpha = 0$, $C = 2$, $K_0 = 0$ and $\theta(0) = 1$

	$b=\frac{1}{3}\log 2$	$b=\frac{1}{4}\log 2$	$b=\frac{1}{5}\log 2$	$b=\frac{1}{6}\log 2$	$b=\frac{1}{10}\log 2$
$(\tau_i.I(\tau_i))$	4.6759 : 1.3116	5.1658 : 1.2381	5.5832 : 1.1914	0 : 0.7418	0 : 0.7752
	8.6561 : 1.5392	9.7814 : 1.4539	10.7534 : 1.3980	7.7656 : 1.2080	9.7219 : 1.1432
	1.9662 : 1.7807	13.5911 : 1.6692	14.9977 : 1.5949	13.0448 : 1.4152	16.3705 : 1.3132
	14.9229 : 1.9995	16.9755 : 1.8614	18.7535 : 1.7685	17.4932 : 1.5907	21.9534 : 1.4524
	17.6530 : 2.2025	20.0857 : 2.0378	22.1932 : 1.9266	21.4683 : 1.7459	26.9204 : 1.5730
	20.2231 : 2.3943	23.0005 : 2.2031	25.4066 : 2.0736	25.1251 : 1.8873	31.4676 : 1.6810
	22.6732 : 2.5779	25.7678 : 2.3601	28.4478 : 2.2125	28.5485 : 2.0188	35.7025 : 1.7797
	25.0300 : 2.7553	28.4191 : 2.5108	31.3530 : 2.3450	31.7914 : 2.1429	39.6921 : 1.8712
	27.3121 : 2.9280	30.9766 : 2.6566	34.1472 : 2.4725	34.8894 : 2.2610	43.4813 : 1.9569
	29.5335 : 3.0970	33.4569 : 2.7986	36.8494 : 2.5960	37.8681 : 2.3745	47.1023 : 2.0376
	31.7048 : 3.2631	35.8726 : 2.9373	39.4737 : 2.7162	40.7466 : 2.4841	50.5789 : 2.1141
	33.8341 : 3.4270	38.2335 : 3.0736	42.0316 : 2.8337	43.5397 : 2.5905	53.9292 : 2.1869
	35.9284 : 3.5893	40.5479 : 3.2079	44.5320 : 2.9489	46.2589 : 2.6942	57.1675 : 2.2563
	37.9929 : 3.7502	42.8221 : 3.3405	46.9826 : 3.0623	48.9138 : 2.7957	60.3051 : 2.3226
	40.0324 : 3.9103	45.0617 : 3.4720	49.3895 : 3.1741	51.5122 : 2.8952	63.3512 : 2.3861
	42.0508 : 4.0697	47.2715 : 3.6024	51.7580 : 3.2847	54.0606 : 2.9931	66.3131 : 2.4470
	44.0513 : 4.2289	49.4554 : 3.7322	54.0927 : 3.3943	56.5646 : 3.0897	69.1972 : 2.5052
	46.0368 : 4.3879	51.6169 : 3.8615	56.3977 : 3.5031	59.0290 : 3.1851	72.0083 : 2.5610
	48.0100 : 4.5471	53.7592 : 3.9905	58.6762 : 3.6113	61.4580 : 3.2795	74.7508 : 2.6143
	49.9730 : 4.7067	55.8849 : 4.1195	60.9316 : 3.7190	63.8553 : 3.3731	77.4280 : 2.6652
	51.9280 : 4.8668	57.9966 : 4.2485	63.1664 : 3.8265	66.2241 : 3.4661	80.0427 : 2.7137
	53.8767 : 5.0275	60.0964 : 4.3779	65.3833 : 3.9339	68.5673 : 3.5586	82.5974 : 2.7599
	55.8207 : 5.1892	62.1864 : 4.5077	67.5845 : 4.0413	70.8876 : 3.6507	85.0937 : 2.8035
	57.7618 : 5.3518	64.2685 : 4.6381	69.7722 : 4.1488	73.1873 : 3.7426	87.5331 : 2.8447
	59.7012 : 5.5156	66.3445 : 4.7692	71.9481 : 4.2566	75.4687 : 3.8343	89.9166 : 2.8834
	61.6403 : 5.6808	68.4159 : 4.9012	74.1143 : 4.3649	77.7338 : 3.9260	92.2448 : 2.9194
	63.5803 : 5.8475	70.4843 : 5.0342	76.2723 : 4.4736	79.9845 : 4.0178	94.5183 : 2.9527
	65.5226 : 6.0158	72.5512 : 5.1684	78.4238 : 4.5831	82.2225 : 4.1098	96.7374 : 2.9784
	67.4682 : 6.1859	74.6181 : 5.3039	80.5704 : 4.6933	84.4495 : 4.2021	98.9488 : 2.4142
	69.4182 : 6.3579	76.6862 : 5.4409	82.7134 : 4.8044	86.6672 : 4.2948	
	71.3738 : 6.5321	78.7570 : 5.5795	84.8544 : 4.9166	88.8770 : 4.3879	
	73.3360 : 6.7086	80.8317 : 5.7199	86.9948 : 5.0299	91.0803 : 4.4817	
	75.3058 : 6.8876	82.9116 : 5.8623	89.1359 : 5.1445	93.2787 : 4.5762	
	77.2844 : 7.0692	84.9980 : 6.0068	91.2791 : 5.2606	95.4734 : 4.6715	
	79.2727 : 7.2536	87.0922 : 6.1536	93.4257 : 5.3783	97.6662 : 4.7607	
	81.2718 : 7.4412	89.1954 : 6.3029	95.5770 : 5.4977	99.9020 : 3.9729	
	83.2828 : 7.6320	91.3090 : 6.4549	97.7349 : 5.6100		
	85.3067 : 7.8263	93.4344 : 6.6098	99.9462 : 4.6388		
	87.3447 : 8.0243	95.5727 : 6.7678			
	89.3980 : 8.2264	97.7260 : 6.9181			
	91.4676 : 8.4327	99.9410 : 5.7311			
	93.5549 : 8.6437				
	95.6610 : 8.8594				
	97.7878 : 9.0647				
	99.9841 : 7.4363				
Revenue (discounted)	371.5616	220.0775	148.0959	108.6965	47.4170
Investment cost (discounted)	39.2258	27.6829	21.7123	19.5772	12.9673
Total profit (discounted)	332.3358	192.3946	126.3837	89.1193	34.4497

Table 10: Impulse Control solutions for b, $T = 100$ and parameter values $\gamma = 0.5$, $r = 0.04$, $\delta = 0.2$, $\beta = 0.2$, $\alpha = 0$, $C = 2$, $K_0 = 0$ and $\theta(0) = 1$.

τ_i	$I(\tau_i)$	$K(\tau_i^-)$	$K(\tau_i^+)$	$\theta(\tau_i^+)$
4.6759	1.3116	0.0000	1.3116	2.0804
8.6561	1.5392	0.5917	1.8351	3.0000
11.9662	1.7807	0.9465	2.2540	3.7648
14.9229	1.9995	1.2478	2.6234	4.4479
17.6530	2.2025	1.5196	2.9623	5.0787
20.2231	2.3943	1.7717	3.2801	5.6725
22.6732	2.5779	2.0094	3.5826	6.2386
25.0300	2.7553	2.2361	3.8733	6.7832
27.3121	2.9280	2.4539	4.1549	7.3104
29.5335	3.0970	2.6645	4.4292	7.8237
31.7048	3.2631	2.8690	4.6976	8.3254
33.8341	3.4270	3.0685	4.9613	8.8173
35.9284	3.5893	3.2636	5.2210	9.3012
37.9929	3.7502	3.4549	5.4776	9.7782
40.0324	3.9103	3.6429	5.7317	10.2495
42.0508	4.0697	3.8280	5.9837	10.7158
44.0513	4.2289	4.0106	6.2342	11.1780
46.0368	4.3879	4.1910	6.4834	11.6368
48.0100	4.5471	4.3693	6.7318	12.0927
49.9730	4.7067	4.5459	6.9797	12.5462
51.9280	4.8668	4.7209	7.2272	12.9979
53.8767	5.0275	4.8945	7.4748	13.4482
55.8207	5.1892	5.0669	7.7226	13.8973
57.7618	5.3518	5.2380	7.9708	14.3458
59.7012	5.5156	5.4082	8.2197	14.7939
61.6403	5.6808	5.5774	8.4695	15.2419
63.5803	5.8475	5.7457	8.7203	15.6902
65.5226	6.0158	5.9133	8.9724	16.1389
67.4682	6.1859	6.0802	9.2260	16.5885
69.4182	6.3579	6.2465	9.4812	17.0390
71.3738	6.5321	6.4121	9.7382	17.4909
73.3360	6.7086	6.5773	9.9972	17.9442
75.3058	6.8876	6.7419	10.2585	18.3993
77.2844	7.0692	6.9061	10.5222	18.8565
79.2727	7.2536	7.0698	10.7885	19.3159
81.2718	7.4412	7.2331	11.0577	19.7778
83.2828	7.6320	7.3959	11.3299	20.2424
85.3067	7.8263	7.5584	11.6055	20.7100
87.3447	8.0243	7.7204	11.8845	21.1809
89.398	8.2264	7.8821	12.1674	21.6553
91.4676	8.4327	8.0433	12.4544	22.1335
93.5549	8.6437	8.204	12.7457	22.6158
95.661	8.8594	8.3642	13.0415	23.1024
97.7878	9.0647	8.5231	13.3263	23.5938
99.9841	7.4363	8.5889	11.7308	24.1012
Revenue (discounted)			371.5616	
Investment cost (discounted)			39.2258	
Total profit (discounted)			332.3358	

Table 11: Impulse Control solutions for $b = \frac{1}{3}\log 2$, $T = 100$ and parameter values $r = 0.04$, $\delta = 0.2$, $\gamma = 0.5$, $\beta = 0.2$, $\alpha = 0$, $C = 2$, $K_0 = 0$ and $\theta(0) = 1$.

τ_i	$I(\tau_i)$	$K(\tau_i^-)$	$K(\tau_i^+)$	$\theta(\tau_i^+)$
5.1658	1.2381	0	1.2381	1.8952
9.7814	1.4539	0.4919	1.6998	2.695
13.5911	1.6692	0.7934	2.0659	3.3552
16.9755	1.8614	1.0499	2.3864	3.9416
20.0857	2.0378	1.2811	2.6784	4.4806
23.0005	2.2031	1.4952	2.9507	4.9857
25.7678	2.3601	1.6965	3.2084	5.4652
28.4191	2.5108	1.888	3.4548	5.9246
30.9766	2.6566	2.0715	3.6924	6.3678
33.4569	2.7986	2.2484	3.9228	6.7976
35.8726	2.9373	2.4197	4.1472	7.2162
38.2335	3.0736	2.5863	4.3668	7.6254
40.5479	3.2079	2.7488	4.5823	8.0264
42.8221	3.3405	2.9077	4.7944	8.4205
45.0617	3.472	3.0634	5.0036	8.8086
47.2715	3.6024	3.2162	5.2105	9.1915
49.4554	3.7322	3.3666	5.4155	9.5700
51.6169	3.8615	3.5147	5.6188	9.9445
53.7592	3.9905	3.6607	5.8209	10.3158
55.8849	4.1195	3.805	6.0219	10.6841
57.9966	4.2485	3.9475	6.2223	11.0500
60.0964	4.3779	4.0885	6.4221	11.4139
62.1864	4.5077	4.2281	6.6217	11.7761
64.2685	4.6381	4.3664	6.8213	12.1369
66.3445	4.7692	4.5035	7.0210	12.4966
68.4159	4.9012	4.6395	7.2210	12.8556
70.4843	5.0342	4.7746	7.4215	13.214
72.5512	5.1684	4.9086	7.6227	13.5722
74.6181	5.3039	5.0418	7.8248	13.9303
76.6862	5.4409	5.1741	8.0280	14.2887
78.7570	5.5795	5.3057	8.2324	14.6475
80.8317	5.7199	5.4365	8.4382	15.0071
82.9116	5.8623	5.5666	8.6456	15.3675
84.9980	6.0068	5.6961	8.8548	15.729
87.0922	6.1536	5.8248	9.0660	16.0919
89.1954	6.3029	5.9529	9.2794	16.4564
91.3090	6.4549	6.0804	9.4951	16.8227
93.4344	6.6098	6.2072	9.7134	17.1909
95.5727	6.7678	6.3334	9.9345	17.5615
97.7260	6.9181	6.4583	10.1472	17.9346
99.9410	5.7311	6.5156	8.9889	18.3185
Revenue (discounted)			220.0775	
Investment cost (discounted)			27.6829	
Total profit (discounted)			192.3946	

Table 12: Impulse Control solutions for $b = \frac{1}{4}\log 2$, $T = 100$ and parameter values $r = 0.04$, $\delta = 0.2$, $\gamma = 0.5$, $\beta = 0.2$, $\alpha = 0$, $C = 2$, $K_0 = 0$ and $\theta(0) = 1$.

τ_i	$I(\tau_i)$	$K(\tau_i^-)$	$K(\tau_i^+)$	$\theta(\tau_i^+)$
5.5832	1.1914	0	1.1914	1.774
10.7534	1.398	0.4236	1.6098	2.4907
14.9977	1.5949	0.6888	1.9393	3.0791
18.7535	1.7685	0.915	2.226	3.5998
22.1932	1.9266	1.1188	2.486	4.0766
25.4066	2.0736	1.3073	2.7273	4.5221
28.4478	2.2125	1.4845	2.9547	4.9437
31.353	2.345	1.6527	3.1714	5.3464
34.1472	2.4725	1.8136	3.3793	5.7338
36.8494	2.596	1.9685	3.5802	6.1084
39.4737	2.7162	2.1182	3.7753	6.4722
42.0316	2.8337	2.2635	3.9654	6.8268
44.532	2.9489	2.4049	4.1514	7.1734
46.9826	3.0623	2.543	4.3338	7.5132
49.3895	3.1741	2.678	4.5131	7.8468
51.758	3.2847	2.8103	4.6899	8.1752
54.0927	3.3943	2.9401	4.8644	8.4988
56.3977	3.5031	3.0678	5.037	8.8184
58.6762	3.6113	3.1934	5.208	9.1343
60.9316	3.719	3.3172	5.3777	9.4469
63.1664	3.8265	3.4393	5.5462	9.7567
65.3833	3.9339	3.5599	5.7138	10.0641
67.5845	4.0413	3.679	5.8808	10.3692
69.7722	4.1488	3.7968	6.0472	10.6725
71.9481	4.2566	3.9134	6.2133	10.9741
74.1143	4.3649	4.0288	6.3793	11.2744
76.2723	4.4736	4.1431	6.5452	11.5736
78.4238	4.5831	4.2564	6.7113	11.8718
80.5704	4.6933	4.3688	6.8776	12.1694
82.7134	4.8044	4.4802	7.0445	12.4665
84.8544	4.9166	4.5907	7.2119	12.7633
86.9948	5.0299	4.7005	7.3801	13.0600
89.1359	5.1445	4.8094	7.5492	13.3569
91.2791	5.2606	4.9176	7.7194	13.6540
93.4257	5.3783	5.0250	7.8908	13.9515
95.5770	5.4977	5.1317	8.0635	14.2498
97.7349	5.6100	5.2372	8.2286	14.5489
99.9462	4.6388	5.2876	7.2826	14.8555
Revenue (discounted)			148.0959	
Investment cost (discounted)			21.7123	
Total profit (discounted)			126.3837	

Table 13: Impulse Control solutions for $b = \frac{1}{5}\log 2$, $T = 100$ and parameter values $r = 0.04$, $\delta = 0.2$, $\gamma = 0.5$, $\beta = 0.2$, $\alpha = 0$, $C = 2$, $K_0 = 0$ and $\theta(0) = 1$.

τ_i	$I(\tau_i)$	$K(\tau_i^-)$	$K(\tau_i^+)$	$\theta(\tau_i^+)$
0	0.7418	0	0.7418	1
7.7656	1.208	0.1570	1.2865	1.8971
13.0448	1.4152	0.4476	1.6390	2.5070
17.4932	1.5907	0.6733	1.9274	3.0209
21.4683	1.7459	0.8703	2.1811	3.4801
25.1251	1.8873	1.0496	2.4121	3.9026
28.5485	2.0188	1.2163	2.6270	4.2981
31.7914	2.1429	1.3734	2.8296	4.6727
34.8894	2.2610	1.5227	3.0224	5.0306
37.8681	2.3745	1.6658	3.2074	5.3747
40.7466	2.4841	1.8036	3.3859	5.7072
43.5397	2.5905	1.9367	3.5589	6.0299
46.2589	2.6942	2.0660	3.7272	6.3440
48.9138	2.7957	2.1917	3.8915	6.6507
51.5122	2.8952	2.3143	4.0524	6.9509
54.0606	2.9931	2.4342	4.2102	7.2453
56.5646	3.0897	2.5516	4.3655	7.5346
59.0290	3.1851	2.6667	4.5184	7.8193
61.4580	3.2795	2.7798	4.6694	8.0999
63.8553	3.3731	2.8909	4.8186	8.3769
66.2241	3.4661	3.0003	4.9663	8.6505
68.5673	3.5586	3.1082	5.1127	8.9212
70.8876	3.6507	3.2145	5.2580	9.1893
73.1873	3.7426	3.3194	5.4023	9.4549
75.4687	3.8343	3.4231	5.5459	9.7185
77.7338	3.9260	3.5255	5.6888	9.9802
79.9845	4.0178	3.6268	5.8312	10.2402
82.2225	4.1098	3.7271	5.9733	10.4987
84.4495	4.2021	3.8263	6.1152	10.7560
86.6672	4.2948	3.9245	6.2570	11.0122
88.8770	4.3879	4.0219	6.3989	11.2675
91.0803	4.4817	4.1183	6.5409	11.5220
93.2787	4.5762	4.2140	6.6832	11.7760
95.4734	4.6715	4.3088	6.8259	12.0295
97.6662	4.7607	4.4024	6.9619	12.2828
99.9020	3.9729	4.4517	6.1988	12.5411
Revenue (discounted)			108.6965	
Investment cost (discounted)			19.5772	
Total profit (discounted)			59.1193	

Table 14: Impulse Control solutions for $b = \frac{1}{6}\log 2$, $T = 100$ and parameter values $r = 0.04$, $\delta = 0.2$, $\gamma = 0.5$, $\beta = 0.2$, $\alpha = 0$, $C = 2$, $K_0 = 0$ and $\theta(0) = 1$.

τ_i	$I(\tau_i)$	$K(\tau_i^-)$	$K(\tau_i^+)$	$\theta(\tau_i^+)$
0	0.7752	0	0.7752	1
9.7219	1.1432	0.1109	1.1987	1.6739
16.3705	1.3132	0.3171	1.4717	2.1347
21.9534	1.4524	0.4818	1.6934	2.5217
26.9204	1.5730	0.6271	1.8866	2.8660
31.4676	1.6810	0.7598	2.0609	3.1812
35.7025	1.7797	0.8835	2.2215	3.4747
39.6921	1.8712	1.0003	2.3713	3.7512
43.4813	1.9569	1.1114	2.5125	4.0139
47.1023	2.0376	1.2179	2.6465	4.2649
50.5789	2.1141	1.3204	2.7743	4.5059
53.9292	2.1869	1.4195	2.8966	4.7381
57.1675	2.2563	1.5157	3.0142	4.9626
60.3051	2.3226	1.6093	3.1273	5.1800
63.3512	2.3861	1.7006	3.2364	5.3912
66.3131	2.4470	1.7897	3.3418	5.5965
69.1972	2.5052	1.8771	3.4437	5.7964
72.0083	2.5610	1.9627	3.5423	5.9912
74.7508	2.6143	2.0468	3.6377	6.1813
77.4280	2.6652	2.1295	3.7300	6.3669
80.0427	2.7137	2.2110	3.8192	6.5481
82.5974	2.7599	2.2913	3.9055	6.7252
85.0937	2.8035	2.3706	3.9888	6.8982
87.5331	2.8447	2.4489	4.0692	7.0673
89.9166	2.8834	2.5263	4.1465	7.2325
92.2448	2.9194	2.6029	4.2208	7.3939
94.5183	2.9527	2.6787	4.2920	7.5515
96.7374	2.9784	2.7537	4.3553	7.7053
98.9488	2.4142	2.7985	3.8134	7.8586
Revenue (discounted)			47.417	
Investment cost (discounted)			12.9673	
Total profit (discounted)			34.4497	

Table 15: Impulse Control solutions for $b = \frac{1}{10}\log 2$, $T = 100$ and parameter values $r = 0.04$, $\delta = 0.2$, $\gamma = 0.5$, $\beta = 0.2$, $\alpha = 0$, $C = 2$, $K_0 = 0$ and $\theta(0) = 1$.

	$C = 4$	$C = 8$	$C = 16$	$C = 32$
$(\tau_i . I(\tau_i))$	5.7915 : 1.8832	8.0844 : 2.4856	11.1517 : 3.3199	15.2866 : 4.4754
	9.6593 : 2.2099	12.7147 : 2.9206	16.6712 : 3.8947	21.8148 : 5.2241
	12.8816 : 2.5607	16.5386 : 3.3546	21.1933 : 4.4297	27.1293 : 5.8789
	15.7638 : 2.8797	19.9372 : 3.7422	25.1901 : 4.8993	31.8052 : 6.4443
	18.4283 : 3.1763	23.0621 : 4.0984	28.8471 : 5.3256	36.0657 : 6.9513
	20.9394 : 3.4571	25.9923 : 4.4325	32.2606 : 5.7215	40.0266 : 7.4169
	23.3358 : 3.7265	28.7755 : 4.7502	35.4889 : 6.0947	43.7577 : 7.8515
	25.6433 : 3.9871	31.4435 : 5.0556	38.5705 : 6.4506	47.3050 : 8.2618
	27.8799 : 4.2412	34.0186 : 5.3513	41.5327 : 6.7926	50.7012 : 8.6525
	30.0590 : 4.4903	36.5173 : 5.6394	44.3957 : 7.1237	53.9701 : 9.0270
	32.1907 : 4.7354	38.9523 : 5.9215	47.1748 : 7.4458	57.1301 : 9.3879
	34.2832 : 4.9775	41.3336 : 6.1989	49.8823 : 7.7606	60.1956 : 9.7373
	36.3429 : 5.2174	43.6692 : 6.4726	52.5280 : 8.0694	63.1782 : 10.0767
	38.3751 : 5.4557	45.9658 : 6.7433	55.1200 : 8.3732	66.0873 : 10.4075
	40.3842 : 5.6929	48.2290 : 7.0118	57.6651 : 8.6731	68.9309 : 10.7307
	42.3740 : 5.9295	50.4635 : 7.2788	60.1692 : 8.9698	71.7156 : 11.0472
	44.3476 : 6.1658	52.6734 : 7.5447	62.6371 : 9.2641	74.4470 : 11.3579
	46.3080 : 6.4022	54.8622 : 7.8101	65.0733 : 9.5565	77.1303 : 11.6634
	48.2575 : 6.6390	57.0331 : 8.0754	67.4816 : 9.8476	79.7695 : 11.9643
	50.1983 : 6.8765	59.1889 : 8.3409	69.8655 : 10.1379	82.3685 : 12.2611
	52.1324 : 7.1149	61.3321 : 8.6072	72.2281 : 10.4280	84.9306 : 12.5542
	54.0615 : 7.3546	63.4651 : 8.8745	74.5721 : 10.7183	87.4588 : 12.8441
	55.9872 : 7.5957	65.5899 : 9.1432	76.9002 : 11.0092	89.9557 : 13.1311
	57.9112 : 7.8386	67.7086 : 9.4136	79.2148 : 11.3012	92.4238 : 13.4156
	59.8346 : 8.0833	69.8230 : 9.6860	81.5182 : 11.5947	94.8653 : 13.6978
	61.7589 : 8.3302	71.9347 : 9.9609	83.8124 : 11.8901	97.2825 : 13.9625
	63.6852 : 8.5795	74.0455 : 10.2384	86.0995 : 12.1878	99.7136 : 12.0324
	65.6147 : 8.8313	76.1570 : 10.5191	88.3813 : 12.4883	
	67.5486 : 9.0860	78.2706 : 10.8031	90.6599 : 12.7920	
	69.4879 : 9.3438	80.3880 : 11.0909	92.9369 : 13.0994	
	71.4338 : 9.6049	82.5104 : 11.3829	95.2143 : 13.4107	
	73.3871 : 9.8696	84.6394 : 11.6793	97.4939 : 13.7081	
	75.3490 : 10.1381	86.7765 : 11.9807	99.8182 : 11.6236	
	77.3205 : 10.4107	88.9230 : 12.2875		
	79.3027 : 10.6877	91.0805 : 12.6000		
	81.2965 : 10.9694	93.2503 : 12.9188		
	83.3031 : 11.2562	95.4341 : 13.2441		
	85.3235 : 11.5483	97.6337 : 13.5562		
	87.3588 : 11.8462	99.8944 : 11.3351		
	89.4101 : 12.1503			
	91.4786 : 12.4609			
	93.5657 : 12.7786			
	95.6724 : 13.1036			
	97.8006 : 13.4128			
Revenue (discounted)	780.7835	769.1875	747.0746	712.6433
Investment cost (discounted)	79.5936	96.8939	120.5584	150.9987
Total profit (discounted)	701.1899	672.2936	626.5162	561.6447

Table 16: Impulse Control solutions for C, $T = 100$ and parameter values $\gamma = 0.5$, $r = 0.04$, $\delta = 0.2$, $b = \frac{1}{2}\log 2$, $\beta = 0.2$, $\alpha = 0$, $K_0 = 0$ and $\theta(0) = 1$.

τ_i	$I(\tau_i)$	$K(\tau_i^-)$	$K(\tau_i^+)$	$\theta(\tau_i^+)$
5.7915	1.8832	0	1.8832	3.0072
9.6593	2.2099	0.8688	2.6444	4.3477
12.8816	2.5607	1.3881	3.2548	5.4644
15.7638	2.8797	1.8289	3.7941	6.4633
18.4283	3.1763	2.2267	4.2897	7.3868
20.9394	3.4571	2.5960	4.7552	8.2570
23.3358	3.7265	2.9445	5.1987	9.0876
25.6433	3.9871	3.2770	5.6256	9.8873
27.8799	4.2412	3.5967	6.0396	10.6624
30.0590	4.4903	3.9060	6.4433	11.4176
32.1907	4.7354	4.2067	6.8387	12.1565
34.2832	4.9775	4.5001	7.2276	12.8817
36.3429	5.2174	4.7873	7.6111	13.5955
38.3751	5.4557	5.0691	7.9903	14.2998
40.3842	5.6929	5.3463	8.3661	14.9961
42.3740	5.9295	5.6194	8.7392	15.6857
44.3476	6.1658	5.8890	9.1103	16.3697
46.3080	6.4022	6.1554	9.4799	17.0491
48.2575	6.6390	6.4191	9.8485	17.7248
50.1983	6.8765	6.6803	10.2166	18.3974
52.1324	7.1149	6.9393	10.5846	19.0677
54.0615	7.3546	7.1963	10.9528	19.7363
55.9872	7.5957	7.4517	11.3216	20.4037
57.9112	7.8386	7.7054	11.6913	21.0705
59.8346	8.0833	7.9578	12.0622	21.7371
61.7589	8.3302	8.2089	12.4347	22.4040
63.6852	8.5795	8.4589	12.8089	23.0716
65.6147	8.8313	8.7079	13.1853	23.7403
67.5486	9.0860	8.9560	13.5640	24.4106
69.4879	9.3438	9.2033	13.9455	25.0827
71.4338	9.6049	9.4498	14.3298	25.7571
73.3871	9.8696	9.6956	14.7174	26.4340
75.3490	10.1381	9.9408	15.1085	27.1140
77.3205	10.4107	10.1853	15.5034	27.7973
79.3027	10.6877	10.4294	15.9024	28.4842
81.2965	10.9694	10.6728	16.3058	29.1752
83.3031	11.2562	10.9158	16.7141	29.8707
85.3235	11.5483	11.1582	17.1275	30.5709
87.3588	11.8462	11.4001	17.5463	31.2762
89.4101	12.1503	11.6415	17.9711	31.9872
91.4786	12.4609	11.8823	18.4021	32.7041
93.5657	12.7786	12.1225	18.8398	33.4274
95.6724	13.1036	12.3620	19.2846	34.1575
97.8006	13.4128	12.5995	19.7126	34.8951
Revenue (discounted)			780.7835	
Investment cost (discounted)			79.5936	
Total profit (discounted)			701.1899	

Table 17: Impulse Control solutions for $C = 4$, $T = 100$ and parameter values $r = 0.04$, $\delta = 0.2$, $b = \frac{1}{2}\log 2$, $\gamma = 0.5$, $\beta = 0.2$, $\alpha = 0$, $K_0 = 0$ and $\theta(0) = 1$.

τ_i	$I(\tau_i)$	$K(\tau_i^-)$	$K(\tau_i^+)$	$\theta(\tau_i^+)$
8.0844	2.4856	0.0000	2.4856	3.8018
12.7147	2.9206	0.9846	3.4129	5.4066
16.5386	3.3546	1.5885	4.1489	6.7318
19.9372	3.7422	2.1025	4.7935	7.9097
23.0621	4.0984	2.5658	5.3813	8.9927
25.9923	4.4325	2.9949	5.9299	10.0082
28.7755	4.7502	3.3986	6.4495	10.9728
31.4435	5.0556	3.7826	6.9468	11.8975
34.0186	5.3513	4.1507	7.4266	12.7899
36.5173	5.6394	4.5056	7.8922	13.6559
38.9523	5.9215	4.8495	8.3463	14.4998
41.3336	6.1989	5.1839	8.7909	15.3251
43.6692	6.4726	5.5101	9.2276	16.1346
45.9658	6.7433	5.8292	9.6579	16.9305
48.2290	7.0118	6.1419	10.0828	17.7149
50.4635	7.2788	6.4491	10.5033	18.4893
52.6734	7.5447	6.7512	10.9203	19.2552
54.8622	7.8101	7.0489	11.3345	20.0138
57.0331	8.0754	7.3425	11.7466	20.7662
59.1889	8.3409	7.6324	12.1572	21.5133
61.3321	8.6072	7.9190	12.5667	22.2561
63.4651	8.8745	8.2026	12.9758	22.9953
65.5899	9.1432	8.4834	13.3849	23.7317
67.7086	9.4136	8.7617	13.7944	24.4660
69.8230	9.6860	9.0376	14.2048	25.1988
71.9347	9.9609	9.3113	14.6165	25.9307
74.0455	10.2384	9.5830	15.0299	26.6622
76.1570	10.5191	9.8527	15.4454	27.3940
78.2706	10.8031	10.1207	15.8635	28.1265
80.3880	11.0909	10.3870	16.2844	28.8603
82.5104	11.3829	10.6517	16.7087	29.5959
84.6394	11.6793	10.9149	17.1368	30.3338
86.7765	11.9807	11.1765	17.5690	31.0744
88.9230	12.2875	11.4367	18.0058	31.8184
91.0805	12.6000	11.6955	18.4477	32.5661
93.2503	12.9188	11.9528	18.8952	33.3181
95.4341	13.2441	12.2087	19.3485	34.0749
97.6337	13.5562	12.4621	19.7872	34.8373
99.8944	11.3351	12.5899	17.6300	35.6208
Revenue (discounted)			769.1875	
Investment cost (discounted)			96.8939	
Total profit (discounted)			672.2936	

Table 18: Impulse Control solutions for $C = 8$, $T = 100$ and parameter values $r = 0.04$, $\delta = 0.2$, $b = \frac{1}{2}\log 2$, $\gamma = 0.5$, $\beta = 0.2$, $\alpha = 0$, $K_0 = 0$ and $\theta(0) = 1$.

τ_i	$I(\tau_i)$	$K(\tau_i^-)$	$K(\tau_i^+)$	$\theta(\tau_i^+)$
11.1517	3.3199	0.0000	3.3199	4.8649
16.6712	3.8947	1.1008	4.4451	6.7778
21.1933	4.4297	1.7993	5.3294	8.3450
25.1901	4.8993	2.3961	6.0974	9.7302
28.8471	5.3256	2.9343	6.7928	10.9976
32.2606	5.7215	3.4320	7.4375	12.1807
35.4889	6.0947	3.8996	8.0445	13.2995
38.5705	6.4506	4.3434	8.6223	14.3675
41.5327	6.7926	4.7679	9.1766	15.3941
44.3957	7.1237	5.1762	9.7118	16.3864
47.1748	7.4458	5.5707	10.2312	17.3495
49.8823	7.7606	5.9533	10.7372	18.2879
52.5280	8.0694	6.3254	11.2321	19.2048
55.1200	8.3732	6.6884	11.7174	20.1031
57.6651	8.6731	7.0431	12.1947	20.9852
60.1692	8.9698	7.3905	12.6651	21.8530
62.6371	9.2641	7.7312	13.1297	22.7084
65.0733	9.5565	8.0658	13.5894	23.5527
67.4816	9.8476	8.3949	14.0450	24.3874
69.8655	10.1379	8.7189	14.4974	25.2135
72.2281	10.4280	9.0382	14.9471	26.0323
74.5721	10.7183	9.3531	15.3949	26.8447
76.9002	11.0092	9.6640	15.8412	27.6516
79.2148	11.3012	9.9711	16.2868	28.4538
81.5182	11.5947	10.2747	16.7321	29.2520
83.8124	11.8901	10.5749	17.1776	30.0472
86.0995	12.1878	10.8720	17.6238	30.8398
88.3813	12.4883	11.1660	18.0714	31.6306
90.6599	12.7920	11.4572	18.5206	32.4203
92.9369	13.0994	11.7456	18.9722	33.2095
95.2143	13.4107	12.0313	19.4263	33.9987
97.4939	13.7081	12.3135	19.8649	34.7888
99.8182	11.6236	12.4796	17.8634	35.5944
Revenue (discounted)			747.0746	
Investment cost (discounted)			120.5584	
Total profit (discounted)			626.5162	

Table 19: Impulse Control solutions for $C = 16$, $T = 100$ and parameter values $r = 0.04$, $\delta = 0.2$, $b = \frac{1}{2}\log 2$, $\gamma = 0.5$, $\beta = 0.2$, $\alpha = 0$, $K_0 = 0$ and $\theta(0) = 1$.

τ_i	$I(\tau_i)$	$K(\tau_i^-)$	$K(\tau_i^+)$	$\theta(\tau_i^+)$
15.2866	4.4754	0.0000	4.4754	6.2979
21.8148	5.2241	1.2128	5.8305	8.5604
27.1293	5.8789	2.0142	6.8860	10.4023
31.8052	6.4443	2.7029	7.7957	12.0228
36.0657	6.9513	3.3250	8.6138	13.4994
40.0266	7.4169	3.9008	9.3673	14.8722
43.7577	7.8515	4.4416	10.0723	16.1653
47.3050	8.2618	4.9546	10.7391	17.3947
50.7012	8.6525	5.4448	11.3748	18.5717
53.9701	9.0270	5.9158	11.9848	19.7046
57.1301	9.3879	6.3703	12.5730	20.7998
60.1956	9.7373	6.8104	13.1425	21.8622
63.1782	10.0767	7.2379	13.6956	22.8959
66.0873	10.4075	7.6542	14.2345	23.9041
68.9309	10.7307	8.0604	14.7609	24.8896
71.7156	11.0472	8.4574	15.2759	25.8547
74.4470	11.3579	8.8461	15.7810	26.8014
77.1303	11.6634	9.2273	16.2771	27.7313
79.7695	11.9643	9.6014	16.7650	28.6460
82.3685	12.2611	9.9692	17.2457	29.5467
84.9306	12.5542	10.3309	17.7196	30.4347
87.4588	12.8441	10.6871	18.1876	31.3109
89.9557	13.1311	11.0381	18.6501	32.1763
92.4238	13.4156	11.3842	19.1077	33.0317
94.8653	13.6978	11.7258	19.5607	33.8778
97.2825	13.9625	12.0624	19.9937	34.7155
99.7136	12.0324	12.2950	18.1799	35.5581
Revenue (discounted)			712.6433	
Investment cost (discounted)			150.9987	
Total profit (discounted)			561.6447	

Table 20: Impulse Control solutions for $C = 32$, $T = 100$ and parameter values $r = 0.04$, $\delta = 0.2$, $b = \frac{1}{2}\log 2$, $\gamma = 0.5$, $\beta = 0.2$, $\alpha = 0$, $K_0 = 0$ and $\theta(0) = 1$.

	$\eta = 0.01$	$\eta = 0.02$	$\eta = 0.03$
$(\tau_i.I(\tau_i))$	5.2730 : 1.7250	6.3504 : 1.9594	7.5126 : 2.2042
	8.9696 : 2.0366	10.4003 : 2.3175	11.902 : 2.6060
	12.0850 : 2.3821	13.8098 : 2.7062	15.5932 : 3.0359
	14.9011 : 2.7029	16.8941 : 3.0676	18.9345 : 3.4366
	17.5308 : 3.0067	19.7779 : 3.4110	22.0629 : 3.8188
	20.0327 : 3.2991	22.5261 : 3.7427	25.0493 : 4.1897
	22.4425 : 3.5837	25.1779 : 4.0670	27.9368 : 4.5542
	24.7835 : 3.8631	27.7594 : 4.3869	30.7539 : 4.9156
	27.0723 : 4.1393	30.2889 : 4.7047	33.5212 : 5.2765
	29.3212 : 4.4136	32.7803 : 5.0219	36.2541 : 5.6390
	31.5397 : 4.6871	35.2443 : 5.3400	38.9647 : 6.0049
	33.7353 : 4.9607	37.6894 : 5.6602	41.6632 : 6.3757
	35.9140 : 5.2353	40.1229 : 5.9836	44.3578 : 6.7529
	38.0810 : 5.5115	42.5509 : 6.3111	47.0561 : 7.1378
	40.2406 : 5.7900	44.9786 : 6.6436	49.7647 : 7.5319
	42.3967 : 6.0713	47.411 : 6.9821	52.4898 : 7.9365
	44.5525 : 6.3560	49.8523 : 7.3274	55.2371 : 8.3532
	46.7110 : 6.6445	52.3066 : 7.6804	58.0124 : 8.7834
	48.8751 : 6.9374	54.7778 : 8.0421	60.8212 : 9.2288
	51.0474 : 7.2352	57.2696 : 8.4135	63.6691 : 9.6913
	53.2301 : 7.5385	59.7857 : 8.7955	66.562 : 10.1729
	55.4257 : 7.8477	62.3297 : 9.1893	69.5058 : 10.6757
	57.6364 : 8.1634	64.9055 : 9.5961	72.5068 : 11.2023
	59.8643 : 8.4863	67.5167 : 10.0172	75.5718 : 11.7556
	62.1118 : 8.8168	70.1673 : 10.4539	78.7082 : 12.3386
	64.3809 : 9.1557	72.8614 : 10.9080	81.9241 : 12.9551
	66.6739 : 9.5037	75.6034 : 11.3811	85.2281 : 13.6094
	68.9931 : 9.8616	78.3978 : 11.8752	88.6304 : 14.3064
	71.3408 : 10.2301	81.2498 : 12.3925	92.1421 : 15.0518
	73.7194 : 10.6101	84.1647 : 12.9354	95.7764 : 15.8177
	76.1314 : 11.0026	87.1483 : 13.5069	99.595 : 12.3688
	78.5796 : 11.4087	90.2071 : 14.1101	
	81.0668 : 11.8296	93.3484 : 14.7483	
	83.5959 : 12.2664	96.5805 : 15.3926	
	86.1701 : 12.7208	99.9593 : 12.1273	
	88.793 : 13.1942		
	91.4683 : 13.6885		
	94.1999 : 14.2053		
	96.9928 : 14.7182		
	99.8973 : 11.8801		
Revenue (discounted)	762.5966	733.2291	701.2148
Investment cost (discounted)	61.1145	56.6083	52.6074
Total profit (discounted)	701.4821	676.6208	648.6074

Table 21: Impulse Control solutions for η, $T = 100$ and parameter values $\gamma = 0.5$, $r = 0.04$, $\delta = 0.2$, $b = \frac{1}{2}\log 2$, $\beta = 0.2$, $\alpha = 0$, $C = 2$, $K_0 = 0$ and $\theta(0) = 1$.

τ_i	$I(\tau_i)$	$K(\tau_i^-)$	$K(\tau_i^+)$	$\theta(\tau_i^+)$
5.2730	1.7250	0.0000	1.725	2.8275
8.9696	2.0366	0.8236	2.4483	4.1086
12.0850	2.3821	1.3130	3.0386	5.1884
14.9011	2.7029	1.7301	3.568	6.1643
17.5308	3.0067	2.1087	4.0611	7.0757
20.0327	3.2991	2.4622	4.5302	7.9428
22.4425	3.5837	2.7977	4.9825	8.7780
24.7835	3.8631	3.1197	5.423	9.5893
27.0723	4.1393	3.4311	5.8548	10.3825
29.3212	4.4136	3.7340	6.2806	11.1620
31.5397	4.6871	4.0300	6.702	11.9308
33.7353	4.9607	4.3202	7.1208	12.6918
35.9140	5.2353	4.6056	7.5381	13.4469
38.0810	5.5115	4.8870	7.955	14.1979
40.2406	5.7900	5.1649	8.3724	14.9463
42.3967	6.0713	5.4398	8.7912	15.6936
44.5525	6.3560	5.7121	9.212	16.4407
46.7110	6.6445	5.9822	9.6356	17.1888
48.8751	6.9374	6.2504	10.0626	17.9388
51.0474	7.2352	6.5168	10.4936	18.6917
53.2301	7.5385	6.7816	10.9293	19.4481
55.4257	7.8477	7.0451	11.3702	20.2091
57.6364	8.1634	7.3072	11.817	20.9752
59.8643	8.4863	7.5681	12.2703	21.7474
62.1118	8.8168	7.8279	12.7308	22.5263
64.3809	9.1557	8.0865	13.199	23.3127
66.6739	9.5037	8.3439	13.6757	24.1074
68.9931	9.8616	8.6002	14.1617	24.9112
71.3408	10.2301	8.8552	14.6576	25.7248
73.7194	10.6101	9.1088	15.1645	26.5492
76.1314	11.0026	9.3609	15.6831	27.3851
78.5796	11.4087	9.6113	16.2144	28.2336
81.0668	11.8296	9.8598	16.7595	29.0956
83.5959	12.2664	10.1062	17.3195	29.9721
86.1701	12.7208	10.3499	17.8958	30.8643
88.793	13.1942	10.5908	18.4896	31.7733
91.4683	13.6885	10.8283	19.1026	32.7005
94.1999	14.2053	11.0618	19.7362	33.6472
96.9928	14.7182	11.2896	20.363	34.6151
99.8973	11.8801	11.3909	17.5756	35.6218
Revenue (discounted)			762.5966	
Investment cost (discounted)			61.1145	
Total profit (discounted)			701.4821	

Table 22: Impulse Control solutions for $\eta = 0.01$, $T = 100$ and parameter values $r = 0.04$, $\delta = 0.2$, $b = \frac{1}{2}\log 2$, $\gamma = 0.5$, $\beta = 0.2$, $\alpha = 0$, $C = 2$, $K_0 = 0$ and $\theta(0) = 1$.

τ_i	$I(\tau_i)$	$K(\tau_i^-)$	$K(\tau_i^+)$	$\theta(\tau_i^+)$
6.3504	1.9594	0	1.9594	3.2009
10.4003	2.3175	0.8717	2.7533	4.6045
13.8098	2.7062	1.3922	3.4023	5.7861
16.8941	3.0676	1.8360	3.9856	6.8550
19.7779	3.4110	2.2388	4.5304	7.8545
22.5261	3.7427	2.6147	5.0500	8.8070
25.1779	4.0670	2.9714	5.5527	9.7260
27.7594	4.3869	3.3135	6.0437	10.6207
30.2889	4.7047	3.6441	6.5267	11.4973
32.7803	5.0219	3.9655	7.0046	12.3608
35.2443	5.3400	4.2793	7.4796	13.2147
37.6894	5.6602	4.5867	7.9536	14.0622
40.1229	5.9836	4.8887	8.4279	14.9055
42.5509	6.3111	5.1860	8.9041	15.7470
44.9786	6.6436	5.4792	9.3832	16.5884
47.411	6.9821	5.7687	9.8664	17.4314
49.8523	7.3274	6.0550	10.3549	18.2775
52.3066	7.6804	6.3382	10.8495	19.1281
54.7778	8.0421	6.6186	11.3514	19.9845
57.2696	8.4135	6.8963	11.8616	20.8481
59.7857	8.7955	7.1713	12.3812	21.7201
62.3297	9.1893	7.4437	12.9112	22.6018
64.9055	9.5961	7.7133	13.4527	23.4945
67.5167	10.0172	7.9800	14.0072	24.3995
70.1673	10.4539	8.2437	14.5758	25.3181
72.8614	10.9080	8.5040	15.1600	26.2518
75.6034	11.3811	8.7606	15.7614	27.2021
78.3978	11.8752	9.0130	16.3817	28.1706
81.2498	12.3925	9.2606	17.0228	29.159
84.1647	12.9354	9.5028	17.6868	30.1692
87.1483	13.5069	9.7386	18.3762	31.2033
90.2071	14.1101	9.9670	19.0936	32.2634
93.3484	14.7483	10.1868	19.8417	33.3521
96.5805	15.3926	10.3954	20.5903	34.4723
99.9593	12.1273	10.4758	17.3652	35.6432
Revenue (discounted)			733.2291	
Investment cost (discounted)			56.6083	
Total profit (discounted)			676.6208	

Table 23: Impulse Control solutions for $\eta = 0.02$, $T = 100$ and parameter values $r = 0.04$, $\delta = 0.2$, $b = \frac{1}{2}\log 2$, $\gamma = 0.5$, $\beta = 0.2$, $\alpha = 0$, $C = 2$, $K_0 = 0$ and $\theta(0) = 1$.

τ_i	$I(\tau_i)$	$K(\tau_i^-)$	$K(\tau_i^+)$	$\theta(\tau_i^+)$
7.5126	2.2042	0	2.2042	3.6037
11.9020	2.6060	0.9162	3.0641	5.1249
15.5932	3.0359	1.4645	3.7681	6.4042
18.9345	3.4366	1.9315	4.4024	7.5622
22.0629	3.8188	2.3548	4.9962	8.6464
25.0493	4.1897	2.7494	5.5644	9.6814
27.9368	4.5542	3.1233	6.1158	10.6821
30.7539	4.9156	3.4815	6.6563	11.6585
33.5212	5.2765	3.8271	7.1901	12.6176
36.2541	5.6390	4.1625	7.7203	13.5647
38.9647	6.0049	4.4894	8.2496	14.5042
41.6632	6.3757	4.8090	8.7802	15.4394
44.3578	6.7529	5.1221	9.3139	16.3733
47.0561	7.1378	5.4295	9.8526	17.3084
49.7647	7.5319	5.7317	10.3977	18.2471
52.4898	7.9365	6.0290	10.9510	19.1916
55.2371	8.3532	6.3215	11.5139	20.1437
58.0124	8.7834	6.6095	12.0881	21.1056
60.8212	9.2288	6.8927	12.6752	22.0790
63.6691	9.6913	7.1711	13.2769	23.0660
66.5620	10.1729	7.4443	13.8951	24.0686
69.5058	10.6757	7.7120	14.5317	25.0889
72.5068	11.2023	7.9736	15.1891	26.1289
75.5718	11.7556	8.2282	15.8697	27.1912
78.7082	12.3386	8.4750	16.5761	28.2782
81.9241	12.9551	8.7129	17.3115	29.3927
85.2281	13.6094	8.9402	18.0795	30.5378
88.6304	14.3064	9.1552	18.8840	31.7170
92.1421	15.0518	9.3557	19.7296	32.9340
95.7764	15.8177	9.5378	20.5867	34.1936
99.5950	12.3688	9.5919	17.1648	35.5170
Revenue (discounted)			701.2148	
Investment cost (discounted)			52.6074	
Total profit (discounted)			648.6074	

Table 24: Impulse Control solutions for $\eta = 0.03$, $T = 100$ and parameter values $r = 0.04$, $\delta = 0.2$, $b = \frac{1}{2}\log 2$, $\gamma = 0.5$, $\beta = 0.2$, $\alpha = 0$, $C = 2$, $K_0 = 0$ and $\theta(0) = 1$.

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